Journal of Computational Mathematics, Vol.14, No.4, 1996, 291–300.

NONOVERLAPPING DOMAIN DECOMPOSITION METHOD WITH MIXED ELEMENT FOR ELLIPTIC PROBLEMS*1)

H.X. Rui
(*Mathematics Department, Shandong University, Jinan*)

Abstract

In this paper we consider the nonoverlapping domain decomposition method based on mixed element approximation for elliptic problems in two dimentional space. We give a kind of discrete domain decomposition iterative algorithm using mixed finite element, the subdomain problems of which can be implemented parallelly. We also give the existence, uniqueness and convergence of the approximate solution.

1. Introduction

Domain decomposition as a new method of computational mathematics, was developed since the development of parallel computers and multiprocessor supercomputers. Using domain decomposition we can decrease the scale of the problem and implement the sub-problems on parallel computer. From a technical point of view most of domain decomposition methods considered so far have been dealing with finite element methods. In [1, 2] Zhang and Huang have given a kind of nonoverlapping domain decomposition procedure with piecewise linear finite element approximation.

Since the advantage of mixed element method in dealing with some engineering problems when accurate approximates to the first derivatives of the solution of the elliptic problem is required, such as numerical simulation in oil recovery, as early as 1988, Glowinski and Wheeler[3] have given a domain decomposition conjugate gradient algorithm with mixed element.

In this paper we give a kind of nonoverlapping domain decomposition algorithm with mixed finite element, which can be implemented in parallel computer. We also give the existence, uniqueness and convergence analysis. Finally we give the numerical examples.

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* Received March 16, 1994.

1) This work is supported by Natural Science Foundation of Shandong Province and National Natural Science Foundation of China.

2. Domain Decomposition Algorithm

Without loss of generality we consider the following problem:

$$\begin{cases} -\triangle u = f, & \text{in}\Omega, \\ \dfrac{\partial u}{\partial n} = 0, & \text{on}\partial\Omega. \end{cases} \tag{1}$$

where $\Omega\subset R^2$ is a bounded domain and can be decomposed into two polygonal domains, n is the unit vector in outer normal direction, $f \in L^2(\Omega)$ satisfying

$$\int_\Omega f dx = 0. \tag{2}$$

We decomposition Ω into nonoverlapping subdomains Ω_1, Ω_2 such that $\bar{\Omega} = \bar{\Omega}_1 \cup \bar{\Omega}_2$ and Ω_1, Ω_2 are two polygonal domains, $\partial\Omega_1 \cap \partial\Omega_2$ is a striaight line. Let

$$\Gamma = \partial\Omega_1 \cap \partial\Omega_2 \cap \Omega; \Gamma_i = \partial\Omega \cap \partial\Omega_i (i = 1, 2).$$

then $\partial\Omega_i = \Gamma \cup \Gamma_i (i = 1, 2)$.

Let $(\cdot,\cdot)$ denote the innerproduct on $L^2(\Omega)$ or $(L^2(\Omega))^2$, $(\cdot,\cdot)_i$ denote the innerproduct on $L^2(\Omega_i)$ or $(L^2(\Omega_i))^2$, $\sigma = -\bigtriangledown u$, then we can derive the mixed formulation of (1): Find $(\sigma, u) \in H^0(\text{div};\Omega) \times L^2(\Omega)$ such that

$$\begin{cases} (\sigma, q) - (\text{div}q, u) = 0, & \forall q \in H^0(\text{div}, \Omega), \\ (\text{div}\sigma, w) = (f, w), & \forall w \in L^2(\Omega), \end{cases} \tag{3}$$

where

$$H(\text{div};\Omega) = \{v : v \in (L^2(\Omega))^2, \text{div}v \in L^2(\Omega)\},$$

$$H^0(\text{div};\Omega) = \{v : v \in H^0(\text{div}, \Omega), v \cdot n|_{\partial\Omega} = 0\}.$$

Under the condition of

$$\int_{\Omega_1} u dx = 0, \tag{4}$$

the problem (3), or (1), has a unique solution.

Let $\Omega_h = \{k\}$ denote the quasi-uniform trianglation of Ω based on the discretization of Ω_1, Ω_2, with elements of size h. We choose $Q_h \times M_h \subset H^0(\text{div}, \Omega) \times L^2(\Omega)$ as the lowest order Raviart-Thomas mixed element space. Then the mixed element solution of problem (3), $(\sigma_h, u_h) \in Q_h \times M_h$, satisfying

$$\begin{cases} (\sigma_h, q) - (\text{div}q, u_h) = 0, & \forall q \in Q_h, \\ (\text{div}\sigma_h, v) = (f, v), & \forall v \in M_h. \end{cases} \tag{5}$$

$$\int_{\Omega_h} u_1 dx = 0. \tag{6}$$