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A COLUMN RECURRENCE ALGORITHM FOR SOLVING LINEAR LEAST SQUARES PROBLEM[*1)]

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Abstract

A new column recurrence algorithm based on the classical Greville method and modified Huang update is proposed for computing generalized inverse matrix and least squares solution. The numerical results have shown the high efficiency and stability of the algorithm.

1. Introduction

Numerical method of a generalized inverse matrix and corresponding with the linear least squares is a standard tool for solving such problems as control, state evaluation and identification. Let A be an $m \times n$ real matrix. A real $n \times m$ matrix G is called the M-P generalized inverse matrix of A if G satisfies the following conditions:

$$\begin{aligned} \text{(I)} \quad AGA &= A, \quad \text{(II)} \quad GAG = G, \\ \text{(III)} \quad (AG)^T &= AG, \quad \text{(IV)} \quad (GA)^T = GA. \end{aligned} \tag{1}$$

Usually, we write G

$$A^+ = G.$$

The linear least square problem is defined as the minimization of the norm of the residual vector

$$\min_x \|Ax - b\|_2^2, \tag{2}$$

where b is an m-vector and x is an n-vector. Thus, the least square solution of the minimum norm of problem (2) is

$$x = A^+ b.$$

One of the major stability indices for computing generalized inverse matrix or linear least squares problem is

$$\kappa(A) = \|A\| \|A^+\|.$$

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An ill-conditioned matrix A, i.e. matrix with large κ, is quite common in control and identification problem[5]. It is thus important to have computational procedures suitable for solving ill-conditioned problem. An excellent survey on linear least square has been given in Björck[1].

Since the Greville scheme[6] is relative simple and is called G-method, it is adopted frequently for computing generalized inverse matrix in some cases. Computational practice and theoretical analysis show, however, when A is an ill-conditioned matrix, that the solution computed by G-method may bear no resemblance to the true solution. On the other hand, modified Huang method, one of the ABS class, may be more stable than that of some classical matrix factorization method[2]. But this method is only fit for solving the problem where $m \le n$.

Our aim of this paper is to describe a new modification of the classical Greville method, which retains the main advantages of the classical scheme but in many cases is more stable.

Throughout this paper, let $\|\cdot\|$ stand for the 2-norm of a matrix or a vector.

2. Greville Method and Its Modification

Let A be a matrix of order $m \times n$ and will be denoted by

$$A = [a_1, a_2, \cdots, a_n] \in R^{m\times n},$$

where $a_i \in R^n$ and $m \ge n$. By convention, we assume rank $(A) = n$.

Denoted by

$$A_1 = [a_1], \quad A_k = [A_{k-1}, a_k] \in R^{m\times k}, \quad k \le n. \tag{3}$$

We have known that G-method is an electable method for computing generalized inverse matrix if A is not too ill-conditioned. The G-method proceeds as follows[6].

G-method:

Set

$$A_1^+ = a_1^T/(a_1^T a_1).$$

For k=2:n Compute

$$d_k = A_{k-1}^+ a_k, \quad c_k = a_k - A_{k-1} d_k;$$

Take

$$y_k^T = \begin{cases} c_k^+ & c_k \ne 0 \\ (1 + d_k^T d_k) d_k^T A_{k-1}^+ & c_k = 0\ . \end{cases}$$

Compute

$$A_k^+ = \begin{bmatrix} A_{k-1}^+ - d_k y_k^T \\ y_k^T \end{bmatrix}.$$

Set

$$A^+ = A_n^+.$$