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DELTA-WAVE IN A SIMPLE 1-D 2×2 HYPERBOLIC PROBLEM*1)

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Abstract

A simple one-dimensional 2×2 hyperbolic system is considered in the paper. The model contains a linear hyperbolic equation, as well as a hyperbolic equation of which the coefficients are about the solution of the linear one. The exact solution is presented and discussed, then numerical experiments are given by TVD (or MmB) type schemes for Riemann problems. From the results, we know that the solutions do have $\delta-$waves for some suitable initial data.

1. Introduction

The classical solution structure has been studied from general hyperbolic system in conservation laws. Mathematicians focused on the existences and uniqueness of solutions and looked for a way to study for practical problems, and with the development of computer sciences, the study has got a great success by using of computational methods, such as finite difference, finite element and spectral methods, to describe natural phenomena.

In recent years, a new singular phenomenon has been discovered, called $\delta-$wave. The first result was presented by Korchinski[1] in 1977, he proved that no classical solutions exist for the following 2×2 system.

$$\begin{cases} u_t + (u^2/2)_x = 0 \\ v_t + (uv)_x = 0\ . \end{cases} \tag{1.1}$$

In 1993, Joseph proved that the viscosity solution contains delta-measures for Riemann problem of the above problem [2].

In 1989, our group began to study for 2-D Riemann problem of 2-D 2×2 nonlinear hyperbolic system in conservation laws on both theoretical analyses and numerical computations,

$$\begin{cases} u_t + (u^2)_x + (uv)_y = 0 \\ v_t + (uv)_x + (v^2)_y = 0 \end{cases} \tag{1.2}$$

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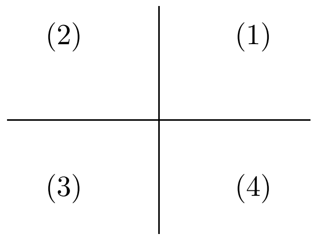
with the 2-D Riemann data

$$(u,v)|_{t=0} = (u_0^i, v_0^i), \quad (i) = 1,2,3,4 \tag{1.3}$$

where (i)-states are described to



Here (1.2) is called 2-D inviscid Burger's equations.

In the processes of the study on the solution structure of the problem, then firstly, we found a new kind of phenomenon by numerical computations for some distributions of Riemann initial data. The phenomenon in numerical results performed that there is a narrow region near shock waves that solution may produce infinity even though the initial data are bounded[3]. The theoretical analyses for (1.2) and (1.3) are given in [4,5]. From the 2-D model, we go back to some 1-D cases, then consider the 1-D 2×2 nonlinear hyperbolic system in conservation laws[4,6], with initial data

$$(u,v)|_{t=0} = (u_0(x), v_0(x)). \tag{1.5}$$

The general model was first presented in [4]. The several special system were proposed and studied in [7] and some results that solutions may produce $\delta-$waves were presented in [8] for 1-D and 2-D hyperbolic systems.

In this paper, we consider the special case of system (1.1). The exact solution is presented and discussed for the Riemann initial data in section 2, then in section 3 the numerical experiments are given for the corresponding Riemann data by TVD (or MmB) schemes. From the exact solutions and numerical solutions, $\delta-$wave do exist in some cases.

2. Exact Solutions

Here we consider the following simple 2×2 hyperbolic system,

$$\begin{cases} u_t + a u_x = 0 \\ v_t + (uv)_x = 0 \end{cases} \tag{2.1}$$

with initial data

$$(u,v)|_{t=0} = (u_0(x), v_0(x)). \tag{2.2}$$

For the first equation of (2.1) and (2.2), the solution can easily be obtained,

$$u(x,t) = u_0(x - at). \tag{2.3}$$