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DIFFERENCE SCHEMES WITH NONUNIFORM MESHES FOR NONLINEAR PARABOLIC SYSTEM*1)

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Abstract

The boundary value problem for the nonlinear parabolic system is solved by the finite difference method with nonuniform meshes. The existence and a priori estemates of the discrete vector solutions for the general difference schemes with unequal meshsteps are established by the fixed point technique. The absolute and relative convergence of the discrete vector solution are justified by a series of a priori estimates. The analysis of mentioned problems are based on the assumption of heuristic character concerning the existence of the unique smooth solution for the original problem of the nonlinear parabolic system.

1. Introduction

1. From the very beginning of sixties to the late eighties, there are many works contributed to the studies of the boundary problems and initial value problems for the ordinary differential equations by the method of difference schemes with nonuniform meshes[1−4]. But it is extremely rare on the works concerning to the analysis of finite difference schemes with nonuniform meshes for the problems of partial differential equations. By using of the difference schemes with nonuniform meshes approximation for the problems of partial differential equations there are many unexpected phenomenon and self-contradictive things both in computations and in analysis.

In this work, we are going to study the difference schemes with nonuniform meshes approximated to the boundary problem for nonlinear parabolic systems of partial differential equations under the assumption of the heuristic character for the existence and uniqueness of the smooth solution of the mentioned problem.

Let us now consider the boundary problem of the nonlinear parabolic systems of second order

$$u_t = A(x,t,u,u_x)u_{xx} + f(x,t,u,u_x), \tag{1}$$

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where $u = (u_1, \cdots, u_m)$ is the m-dimensional unknown vector function ($m \geq 1$), $A(x,t,u,p)$ is a $m \times m$ matrix function, $f(x,t,u,p)$ is a m-dimensional vector function and $u_t = \frac{\partial u}{\partial t}$, $u_x = \frac{\partial u}{\partial x}$ and $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ are the corresponding m-dimensional vector derivatives. The coefficient matrix $A(x,t,u,p)$ is positive definite, hence the system is parabolic. Let us consider in the rectangular domain $Q_T = \{0 \leq x \leq l, \quad 0 \leq t \leq T\}$ with the given positive constants $l > 0$ and $T > 0$, the boundary problem for the nonlinear parabolic system (1) of partial differential equations with the boundary conditions

$$\begin{aligned} u(0,t) &= \psi_0(t), \\ u(l,t) &= \psi_1(t) \end{aligned} \tag{2}$$

and the initial condition

$$u(x,0) = \phi(x), \tag{3}$$

where $\psi_0(t)$, $\psi_1(t)$ and $\phi(x)$ are given m-dimensional vector functions of variables $t \in [0, \ T]$ and $x \in [0, \ l]$ respectively.

In this work the existence and the estimates of the discrete solutions for the finite difference schemes with nonuniform meshes are established by the fixed point technique[5]. The absolute and relative convergence of the discrete solutions of difference schemes with unequal meshsteps are justified by means of a series of a priori estimates. It is notice that in the present, the existence of the unique smooth solution for original problem of the nonlinear parabolic system is assumed to be valid. This is the fundamental assumption of heurustic character in the present study.

In the present investigation for the difference schemes with nonuniform meshes we are repeatedly using the methods and treatments similar to the study of analogous problems for the cases of difference schemes with equal meshstep.

2. Now suppose that for the boundary problem (2) and (3) of the nonlinear parabolic system (1) of second order, the following conditions are fulfilled.

(I) The boundary problem (2) and (3) for the nonlinear parabolic system (1) has a unique smooth m-dimensional vector solution $u(x,t) \in C_{x,t}^{(4,2)}(Q_T)$.

(II) The coefficient matrix $A(x,t,u,p)$ is positively definite, that is, there is a positive constant $\sigma_0 > 0$, such that for any $\xi \in R^m$,

$$(\xi, \ \ A(x,t,u,p)\xi) \geq \sigma_0 |\xi|^2, \tag{4}$$

where $(x,t) \in Q_T$ and $u, \ \ p \in R^m$.

(III) The m-dimensional vector function $f(x,t,u,p)$ and the $m \times m$ matrix function $A(x,t,u,p)$ are continuous with respect to variables $(x,t) \in Q_T$ and continuously differentiable with respect to m -dimensional vector variables $u, \ \ p \in R^m$.

(IV) The boundary vector functions $\psi_0(t)$ and $\psi_1(t)$ are continuously differentiable with respect to $t \in [0, \ T]$. The initial vector function $\phi(x)$ is continuously differentiable