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THE LARGE TIME CONVERGENCE OF SPECTRAL METHOD FOR GENERALIZED KURAMOTO-SIVASHINSKY EQUATIONS*1)

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Abstract

In this paper we use the spectral method to analyse the generalized Kuramoto-Sivashinsky equations. We prove the existence and uniqueness of global smooth solution of the equations. Finally, we obtain the error estimation between spectral approximate solution and exact solution on large time.

1. Introduction

The following nonlinear evolution equation

$$\phi_t + \frac{1}{2}\phi_x^2 + \nu\phi + \alpha\phi_{xx} + \beta\phi_{xxx} + \gamma\phi_{xxxx} = 0, \alpha, \beta, \nu > 0, \tag{1.1}$$

arises in different problem of physics. Kuramoto[1] derived it for the study of dissipative structure of reaction-diffusion. Sivashinsky[2] derived it independently in studying the propagation of a flame front in case of mild combustion. Then it was obtained in bifurcating solution of the Navier-Stokes equation[3] and in viscous film flow[4]. In [5-8], the bifurcating solution, universal attractor was studed for (1.1). The paper [9] proposed a class of generalized KS equations. In [10-11], for some equations of the generalized KS type, the author studed the existence, the uniqness of the global smooth solution, the asymptotic behaviour at $t \to \infty$, the structure of traveling wave solution and approximate solution by Lie group and infinitesimal transformation.

In infinite dimensional dynamical system, as the Navier-Stokes equation, the Boussinesq-Newton equation produced many important physical phenomena, for example, the universal attractor, the chaos etc. It was need to study the large time new computational methods and convergence proof. Similar , it is need to study the large time convergence and error estimation of the approximate solution for KS equation.

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In this paper we use the spectral method to analyse the periodic initial value problem of the generalized KS equations of the form

$$\begin{cases} u_t+\alpha u_{xx}+\beta u_{xxx}+\gamma u_{xxxx}+f(u)_x \\ \qquad\qquad +\phi(u)_{xx}=g(u)+h(x,t), \qquad t>0, x\in R, \alpha,\gamma>0, \\ u\mid_{t=0}=u_0(x), x\in R, u(x+2\pi,t)=u(x,t), \qquad t\geq 0, x\in R. \end{cases} \tag{1.2}$$

We prove the convergence of the approximate solution and estimate the error for $t\geq 0$. If differentiating (1.1) with respect to x and setting $u=\phi_x$, then (1.1) is particular form of (1.2).

2. On Large Time Priori Error of Approximate Solution, the Existence and the Uniqueness of Global Solution

Let $\Omega=[0,2\pi]$, $L^2(\Omega)$ denote the set of all square integrable functions with the inner product $(u,v)=\int_0^{2\pi}u(x)v(x)dx$ and the norm $\|u\|^2=(u,u)$. Let $L^\infty(\Omega)$ denote the Lebesgue space with the norm $\|u\|_{L^\infty}=\text{ess sup}_{x\in\Omega}\mid u(x)\mid$ and H_p^m denote the periodic Sobolev space with the norm $\|u\|_m=(\sum_{|\alpha|\leq m}\|D^\alpha u\|^2)^{\frac{1}{2}}$. We define $L^2(R^+;H_p^m(\Omega))=\{u(x,t)\in H_p^m(\Omega)|\int_0^{+\infty}\|u(x,t)\|_m^2dt<+\infty\}$ and $L^\infty(R^+;H_p^m(\Omega))=\{u(x,t)\in H_p^m(\Omega)$ $|\sup_{0\leq t<+\infty}\|u(x,t)\|_m<+\infty\}$. Let $v_j=\frac{1}{\sqrt{2\pi}}e^{ijx}$ are basis functions, $S_N=\text{Span}\{v_j(x),$ $-N\leq j\leq N\}$.

We construct an approximate solution for periodic initial value problem (1.2) as following: find $u_N\in S_N$, such that

$$(u_{Nt}+\alpha u_{Nxx}+\beta u_{Nxxx}+\gamma u_{Nxxxx}+f(u_N)_x + \phi(u_N)_{xx}-g(u_N)-h(x,t),v_j)=0, \tag{2.1}$$

$$u_N\mid_{t=0}=u_{0N}(x)=F_Nu_0(x), \qquad -N\leq j\leq N, \tag{2.2}$$

where F_N is orthogonal projecting operator from $L^2(\Omega)$ to S_N in the inner product of $L^2(\Omega)$.

Lemma 1. *If the following conditions are satisfied*

1. $\phi'(u)\leq\phi_0, \phi_0>0$;
2. $g(0)=0,\ g'(u)\leq g_0,\ g_0<-\frac{1}{2}(\alpha+\phi_0+1),\ \gamma>\frac{1}{2}(\alpha+\phi_0)$;
3. $h(x,t)\in L^2(Q_\infty),\ u_0(x)\in L^2(\Omega),\ Q_\infty=\Omega\times R^+$,

then for the solution of problem (2.1)-(2.2), *we have*

$$\|u_N(.,t)\|^2\leq e^{2(g_0+\frac{1}{2}(\alpha+\phi_0+1))t}\|u_0(x)\|^2+\|h\|^2_{L^2(Q_\infty)}, \qquad 0\leq t<+\infty,$$

$$\int_0^{+\infty}\|u_N(.,t)\|^2\,dt\leq\frac{1}{2|g_0+\frac{1}{2}(\alpha+\phi_0+1)|}(\|h\|^2_{L^2(Q_\infty)}+\|u_0\|^2),$$

$$\int_0^{+\infty}\|u_{Nxx}(.,t)\|^2\,dt\leq E_1,$$