Journal of Computational Mathematics, Vol.15, No.3, 1997, 279–287.

SPLITTING A CONCAVE DOMAIN TO CONVEX SUBDOMAINS*[1)]

H.C. Huang W.M. Xue
(*Department of mathematics, Hong Kong Baptist University, Kowloon, Hongkong*)
S. Zhang
(*ICMSEC, Chinese Academy of Sciences, Beijing, China*)

Abstract

We will study the convergence property of Schwarz alternating method for concave region where the concave region is decomposed into convex subdomains. Optimality of regular preconditioner deduced from Schwarz alternating is also proved. It is shown that the convergent rate and the condition number are independent of the mesh size but dependent on the relative geometric position of subdomains. Special care is devoted to non-uniform meshes, exclusively, local properties like the shape regularity of the finite elements are utilized.

1. Introduction

Turning large scale problem to small scale subproblems and regularizing irregular problem are two main subjects of domain decomposition. In regularization, regularizing irregular region is of first importance. Irregularity often means concavity, for example, L-shaped, T-shaped and C-shaped domains are irregular domains. In this paper, we will study domain decomposition method for elliptic problems defined on irregular region.

Schwarz alternating method is the basis of almost all domain decomposition method developed. Other methods are variations of it in nature and it was originally designed to regularize concave domain. When the domain is regularized, various fast algorithms may be used.

For continuous problem, [1] has given a complete theory. When subdomains have uniform overlap, Schwarz method has been studied sufficiently for discrete problem. When the domain is concave, the subdomains will have not a uniform overlap. Schwarz method has not been understood clearly for discrete problem. [2] and [3] studied this problem in special cases. We will study this problem generally. We will show that the convergence rate is independent of the mesh size but dependent on the relative position of subdomains. Some optimal preconditioners derived from Schwarz method will be studied as well. Triangulation will not be supposed to be quasi-uniform but it should be local shape regular.

* Received May 6, 1995.
[1)]This work was supported by Research Grants Council of the Hong Kong UGC.

2. On Some Projection Operators

Let $\Omega \subset R^2$ be a concave polygonal region

$$Lu = -\sum_{i,j=1}^{2} \frac{\partial u}{\partial x_i}\Big(a_{ij}\frac{\partial u}{\partial x_j}\Big) + cu$$

be an elliptic operator defined on it, here, $(a_{i,j})_{i,j=1,2}$ is symmetric positive definite and bounded from above and below on Ω, $c \geq 0$.

$$\begin{cases} a(u,v) = (f,v), \quad v \in H_0^1(\Omega) \\ u \in H_0^1(\Omega) \end{cases} \tag{2.1}$$

is the variational form of the boundary value problem, the bilinear form

$$a(u,v) = \int_\Omega \Big[\sum_{i,j=1}^{2} a_{ij}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j} + cu\dot{v}\Big].$$

For convenience we only discuss the homogeneous Dirichlet boundary value problem here. The norm in $H_0^1(\Omega)$ introduced by $a(\cdot,\cdot)$ is equivalent to the original one. $H_0^1(\Omega)$ will be treated as a Hilbert space with inner product $a(\cdot,\cdot)$ in the following.

(2.1) is discretized by finite element method. Triangulation and linear continuous element will be discussed. The triangulation is supposed to be local shape regular. The diameter of an element, if the element does not intersect with the boundary of Ω, does not exceed the product of a constant and the distance from the element to the boundary of Ω.

$S_0^h(\Omega)$ represents the finite element space.

The discrete form of (2.1) is

$$\begin{cases} a(u,v) = (f,v), \qquad v \in S_0^h(\Omega) \\ u \in S_0^h(\Omega) \end{cases} \tag{2.2}$$

Ω_1 and Ω_2 are two convex subdomains of Ω, $\Omega = \Omega_1 \cup \Omega_2$, $\Omega_1 \cap \Omega_2 \neq \emptyset$. The boundaries of Ω_1 and Ω_2 coincide with the finite element triangulation. $\Gamma_1 = \partial\Omega_1 \cap \Omega_2$,$\Gamma_2 = \partial\Omega_2 \cap \Omega_1$, $\bar{\Gamma}_1 \cap \bar{\Gamma}_2$ is the concave point. Here, we suppose Γ_1 and Γ_2 are straight lines and the angle between Γ_1 and Γ_2 is θ.

$$S_0^h(\Omega_1) = S_0^h(\Omega) \cap H_0^1(\Omega_1), \quad S_0^h(\Omega_2) = S_0^h(\Omega) \cap H_0^1(\Omega_2). \tag{2.3}$$

Figure 1 Figure 2