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SPECTRAL-DIFFERENCE METHOD FOR TWO-DIMENSIONAL COMPRESSIBLE FLUID FLOW*[1]

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Abstract

We develop a combined Fourier spectral-finite difference method for solving 2-dimensional, semi-periodic compressible fluid flow problem. The error estimation, as well as the convergence rate, is presented.

Key Words: Spectral-difference method, compressible fluid flow, error estimation.

1. Introduction

We consider the following compressible flow equations:

$$\begin{cases} \dfrac{\partial u_i}{\partial t} + (u\cdot\nabla)u_i - \dfrac{1}{\rho}\dfrac{\partial}{\partial x_i}(\kappa\nabla\cdot u) - \dfrac{1}{\rho}\sum\limits_{j=1}^{2}\dfrac{\partial}{\partial x_j}\Big[\nu\Big(\dfrac{\partial u_i}{\partial x_j}+\dfrac{\partial u_j}{\partial x_i}\Big)\Big] + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x_i} = f_i,\ i=1,2, \\ \dfrac{\partial T}{\partial t} + (u\cdot\nabla)T - \dfrac{1}{\rho T S_T}(\nabla\cdot\mu\nabla)T - \dfrac{\nu}{2\rho T S_T}\sum\limits_{i,j=1}^{2}\Big(\dfrac{\partial u_j}{\partial x_i}+\dfrac{\partial u_i}{\partial x_j}\Big)^2 \\ \qquad -\dfrac{\kappa}{\rho T S_T}(\nabla\cdot u)^2 - \dfrac{\rho S_\rho}{S_T}(\nabla\cdot u) = 0, \\ \dfrac{\partial \rho}{\partial t} + (u\cdot\nabla)\rho + \rho(\nabla\cdot u) = 0, \end{cases} \tag{1.1}$$

where u is the velocity, $u=(u_1,u_2)^*$, T is the absolute temperature, $\nu(T,\rho)$ is the viscous coefficient, $\kappa(T,\rho)=\nu'(T,\rho)-\dfrac{2}{3}\nu(T,\rho)$ with $\nu'(T,\rho)$ being the second viscous coefficient. $\mu(T,\rho)$ is the coefficient of heat conduction, $S(T,\rho)$ is the entropy, $S_T=\dfrac{\partial S}{\partial T}$, $S_\rho=\dfrac{\partial S}{\partial \rho}$.

Under certain conditions, Tani[2] proved that the first boundary problem of (1.1) possesses unique local classical solution. Towards the numerical solution of this problem, the classical difference method is convenient, but it has lower approximate accuracy. Finite element method is particularly suitable for problems with irregular domains.

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In the past sixteen years, spectral method for P.D.E. has been developed rapidly[6−10]. In [7], spectral method was used to solve (1.1), but only for periodical problems.

For many practical problems, the boundary conditions are neither fully periodic nor fully nonperiodic. An effective strategy to deal with such problems is to combine the Fourier spectral method with finite difference method or finite element method. In [6], Guo and Cao established spectral-finite element scheme for solving such semi-periodic problems, but it lacks boundary errors analysis. This paper is devoted to a Fourier spectral-finite difference method to solve such problems, we strictly analyse the errors induced by the initial values and boundary conditions.

Let $\Omega = I \times I^*$, $I = (0,1)$ and $I^* = (0,2\pi)$, we consider the solution of (1.1) in the domain $\Omega \times [0,t_0]$. We suppose that all functions in (1.1) have the periodicity 2π in the x_2 direction, and that u, T satisfy the first kind boundary conditions. These mean that

$$\begin{cases} \eta|_{x_2=0} = \eta|_{x_2=2\pi}, & \forall (x_1,t) \in I \times [0,t_0], \quad \eta = u, T, \rho, \\ u|_{x_1\in\partial I} = g_1(x_2,t). \quad T|_{x_1\in\partial I} = g_2(x_2,t), & \forall (x_2,t) \in I^* \times [0,t_0]. \end{cases} \tag{1.2}$$

Besides, we assume that the initial values of (1.1) are the following,

$$\eta|_{t=0} = \eta_0, \qquad \eta = u, T, \rho. \tag{1.3}$$

To avoid "negative density"(i.e. $\rho < 0$), which is likely caused by the round off errors during the computations, and which generates a non-physical solution and instablize the computations, we adopt the idea of Guo Ben-yu[5−7], i.e., we seek $\varphi = \ln\rho$ by (1.1) instead of calculating ρ directly. besides, we assume the fluid satisfies the following state equation, $p = R\rho T$, where R is a positive constant. Consequently (1.1) can be rewritten into the following form,

$$\begin{cases} \dfrac{\partial u_i}{\partial t} + (u\cdot\nabla)u_i - \mathrm{e}^{-\varphi}\dfrac{\partial}{\partial x_i}(\kappa\nabla\cdot u) - \mathrm{e}^{-\varphi}\displaystyle\sum_{j=1}^{2}\dfrac{\partial}{\partial x_j}\Big[\nu\Big(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\Big)\Big] \\ \qquad +R\dfrac{\partial T}{\partial x_i} + RT\dfrac{\partial \varphi}{\partial x_i} = f_i, \qquad i = 1,2, \\ \dfrac{\partial T}{\partial t} + (u\cdot\nabla)T - \mathrm{e}^{-\varphi}T^{-1}S_T^{-1}(\nabla\cdot\mu\nabla)T - \dfrac{1}{2}\nu\mathrm{e}^{-\varphi}T^{-1}S_T^{-1}\displaystyle\sum_{i,j=1}^{2}\Big(\dfrac{\partial u_j}{\partial x_i} + \dfrac{\partial u_i}{\partial x_j}\Big)^2 \\ \qquad -\kappa\mathrm{e}^{-\varphi}T^{-1}S_T^{-1}(\nabla\cdot u)^2 - S_\varphi S_T^{-1}(\nabla\cdot u) = 0, \\ \dfrac{\partial\varphi}{\partial t} + (u\cdot\nabla)\varphi + (\nabla\cdot u) = 0, \end{cases} \tag{1.4}$$

We suppose ν, μ, κ and S are sufficiently smooth for each of their variables, and there exist positive constants B_0, B_1, B_2, ν_0, ν_1, μ_0, μ_1, κ_1, A_0, A_1, S_0, S_1, S_2, Φ_0 and Φ_1, such that for $B_0 < T < B_1$ and $|\varphi| \le B_2$,

$$\begin{cases} \nu_0 < \nu < \nu_1, \ \mu_0 < \mu < \mu_1, \ |\kappa| < \kappa_1, \ \min(2\kappa + 3\nu, \nu) > A_0, \\ S_0 < S_T < S_1, \ |S_\varphi| < S_2, \ \Phi_0 < \mathrm{e}^{-\varphi} < \Phi_1, \\ \Big|\dfrac{\partial\eta}{\partial z}\Big| \le A_1, \quad \text{where} \ \ \eta = \nu, \kappa, \mu, S_T, S_\varphi, \quad z = T, \varphi. \end{cases} \tag{1.5}$$