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CONTACT ALGORITHMS FOR CONTACT DYNAMICAL SYSTEMS*

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Abstract

In this paper, we develop a general way to construct contact algorithms for contact dynamical systems. Such an algorithm requires the corresponding step-transition map preserve the contact structure of the underlying contact phase space. The constructions are based on the correspondence between the contact geometry of $\mathbf{R}^{2n+1}$ and the conic symplectic one of $\mathbf{R}^{2n+2}$ and therefore, the algorithms are derived naturally from the symplectic algorithms of Hamiltonian systems.

Key words: Contact algorithms, contact systems, conic symplectic geometry, generating functions.

1. Introduction

Contact structure is an analog of a symplectic one for odd-dimensional manifolds, it stems from manifolds of contact elements of configuration spaces in mechanics and, therefore, it is also of basic importance in physical and engineering sciences. We apply, in this paper, the ideas of preserving Lie group and Lie algebra structure of dynamical systems in constructing symplectic algorithms for Hamiltonian systems to the study of numerical algorithms for contact dynamical systems and present so-called contact algorithms, i.e., algorithms preserving contact structures, for solving numerically contact systems.

A contact structure on a manifold is defined as a nondegenerate field of tangent hyperplanes and, therefore, it is determined by a differential 1-form, uniquely up to an everywhere non-vanishing multiplier function, such that the zero set of the 1-form at a point on the manifold is the tangent hyperplane of the field at the point. So, contact structures occur only on manifolds of odd-dimensions. In this paper, we simply consider the Euclidean space $\mathbf{R}^{2n+1}$ of $2n+1$ dimensions as our basic manifold with the contact structure given by the normal form

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[1)] Professor K. Feng was the former director of Computing Center (now it is renamed ICMSEC) of Chinese Academy of Sciences. This paper is arranged by Zai-jiu Shang, based on the notes of Professor K. Feng.

$$\alpha = \sum_{i=1}^{n} x_i dy_i + dz =: xdy + dz = (0, x^T, 1) \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix}, \tag{1.1}$$

here we have used 3-symbol notation to denote the coordinates and vectors on $\mathbf{R}^{2n+1}$

$$x = (x_1, \cdots, x_n)^T, \ y = (y_1, \cdots, y_n)^T, \ z = (z). \tag{1.2}$$

A contact dynamical system on $\mathbf{R}^{2n+1}$ is governed by a contact vector field $f = (a^T, b^T, c^T)$: $\mathbf{R}^{2n+1} \to \mathbf{R}^{2n+1}$ through equations

$$\dot{x} = a(x, y, z), \quad \dot{y} = b(x, y, z), \quad \dot{z} = c(x, y, z), \quad \dot{} =: \frac{d}{dt}, \tag{1.3}$$

where the contactivity condition of the vector field f is

$$L_f \alpha = \lambda_f \alpha \tag{1.4}$$

with some function $\lambda_f : \mathbf{R}^{2n+1} \to \mathbf{R}$, called the multiplier of f. In (1.4), $L_f\alpha$ denotes the Lie derivative of α with respect to f and is usually calculated by the formula[10]

$$L_f \alpha = i_f d\alpha + d i_f \alpha. \tag{1.5}$$

It is easy to show from (1.4) and (1.5) that to any contact vector field f on $\mathbf{R}^{2n+1}$, there corresponds a function $K(x, y, z)$, called contact Hamiltonian, such that

$$a = -K_y + K_z x, \quad b = K_x, \quad c = K - x^T K_x =: K_e. \tag{1.6}$$

In fact, (1.6) represents the general form of a contact vector field. Its multiplier, denoted as λ_K from now, is equal to K_z.

A contact transformation g is a diffeomorphism on $\mathbf{R}^{2n+1}$

$$g : \begin{pmatrix} x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} \hat{x}(x, y, z) \\ \hat{y}(x, y, z) \\ \hat{z}(x, y, z) \end{pmatrix}$$

conformally preserving the contact structure, i.e., $g^*\alpha = \mu_g \alpha$, that means

$$\sum_{i=1}^{n} \hat{x}_i d\hat{y}_i + d\hat{z} = \mu_g \Big(\sum_{i=1}^{n} x_i dy_i + dz \Big) \tag{1.7}$$

for some everywhere non-vanishing function $\mu_g : \mathbf{R}^{2n+1} \to \mathbf{R}$, called the multiplier of g. The explicit expression of (1.7) is

$$(0, \hat{x}^T, 1) \begin{pmatrix} \hat{x}_x & \hat{x}_y & \hat{x}_z \\ \hat{y}_x & \hat{y}_y & \hat{y}_z \\ \hat{z}_x & \hat{z}_y & \hat{z}_z \end{pmatrix} = \mu_g (0, x^T, 1).$$