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THE LARGE TIME CONVERGENCE OF SPECTRAL METHOD FOR GENERALIZED KURAMOTO-SIVASHINSKY EQUATION (II)[*1)]

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Abstract

In this paper, we study fully discrete spectral method and long time behavior of solution of generalized Kuramoto-Sivashinsky equation with periodic initial condition. We prove that the large time error estimation for fully discrete solution of spectral method. We prove the existence of approximate attractors $\mathcal{A}_N$, $\mathcal{A}_N^k$ respectively and $d(\mathcal{A}_N, \mathcal{A}) \to 0$, $d(\mathcal{A}_N^k, \mathcal{A}) \to 0$.

Key words: Kuramoto-Sivashinsky equation, large time convergence, Approximate attractor, Upper semicontinuity of attractors.

1. Introduction

In the paper [1], we studed the generalized Kuramoto-Sivashinsky equation

$$u_t + \gamma u_{xxxx} + \beta u_{xxx} + \alpha u_{xx} + f(u)_x + \phi(u)_{xx} = g(u) + h(x,t). \tag{1.1}$$

We proved the existence and uniqueness of global solution for periodic initial problem and gave the large time error estimation for the solution of continuous spectral method.

The aim of this paper is to study fully discrete spectral method and the long time behavior of the solution of this system. In §1 we given the large time error estimation for fully discrete solution of spectral method. In §2 we prove the existence of approximate attractors $\mathcal{A}_N$, $\mathcal{A}_N^k$ and in §3 we prove the convergence of approximate attractors $d(\mathcal{A}_N, \mathcal{A})$, $d(\mathcal{A}_N^k, \mathcal{A}) \to 0$.

2. The Large Time Error Estimation of Fully Discrete Approximate Solution

For the problem (1.1), we construct the following fully discrete approximate spectral scheme

$$\Big(\frac{1}{k}(u_N^n - u_N^{n-1}) + \alpha u_{Nxx}^n + \beta u_{Nxxx}^n + \gamma u_{Nxxxx}^n + f(u_N^n)_x + \phi(u_N^n)_{xx}$$

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$$- g(u_N^n) - h(x,t_n), \chi\Big) = 0, \quad \forall \chi \in S_N, \tag{2.1}$$

$$u_N^0 = u_{0N} = F_N u_0, \tag{2.2}$$

where the k is step size of time, F_N is the orthogonal projective operator from $L^2(\Omega)$ to S_N.

Lemma 1. *If $f(t)$, $f'(t) \in L^2(R^+)$, let $f_n = f(nk)$, $k > 0$, then*

$$k \sum_{n=1}^{\infty} f_n^2 \le (1+k) \int_0^{+\infty} |f|^2 dt + k \int_0^{+\infty} |f'|^2 dt. \tag{2.3}$$

Proof. Using the integration by parts

$$\begin{aligned} kf_n^2 &= \int_{t_{n-1}}^{t_n} (t - t_{n-1}) \frac{d}{dt} f^2 dt + \int_{t_{n-1}}^{t_n} f^2 dt = 2 \int_{t_{n-1}}^{t_n} (t - t_{n-1}) f f' dt + \int_{t_{n-1}}^{t_n} f^2 dt \\ &\le \int_{t_{n-1}}^{t_n} (t - t_{n-1})(f^2 + f'^2) dt + \int_{t_{n-1}}^{t_n} f^2 dt \\ &\le (1+k) \int_{t_{n-1}}^{t_n} f^2 dt + k \int_{t_{n-1}}^{t_n} f'^2 dt. \end{aligned} \tag{2.4}$$

Summing up for n in both sides of (2.4), we obtain the conclusion of Lemma.

Now we make priori estimates for the solution of (2.1)–(2.2).

Lemma 2. *Under the conditions of Lemma 1 of* [1] *and assume $h_t \in L^2(Q_\infty)$, then we have the estimates for the solution of* (2.1)–(2.2)

$$\|u_N^n\|^2 \le \frac{1}{(1+\lambda k)^n} \|u_0\|^2 + (1+k) \int_0^{+\infty} \|h(t)\|^2 dt + k \int_0^{+\infty} \|h_t(t)\|^2 dt \le C^*, \tag{2.5}$$

$$k \sum_{n=1}^{\infty} \|D_x^j u_N^n\|^2 \le \tilde{C}_j \Big(\|u_0\|^2 + (1+k) \int_0^{+\infty} \|h(t)\|^2 dt + k \int_0^{+\infty} \|h_t(t)\|^2 dt \Big) = C_j^*, \quad 0 \le j \le 2, \tag{2.6}$$

where $\lambda = -2\Big[g_0 + \dfrac{1}{2}(\alpha + \phi_0 + 1)\Big] > 0$, C^*, C_j^*, $0 \le j \le 2$ are constants independent of N.

Lemma 3. *If the conditions of Lemma* 3 *of* [1] *and Lemma* 2 *are satisfied and assume $h_{xt} \in L^2(Q_\infty)$, then we have*

$$\begin{aligned} \|u_{Nx}^n\|^2 \le{}& \frac{1}{(1-2kg_0)^n} \|u_{0x}\|^2 + C\Big[(1+k) \int_0^{+\infty} \|h_x(t)\|^2 + k \int_0^{+\infty} \|h_{xt}(t)\|^2 dt\Big] \\ &+ Ck \sum_{j=1}^{\infty} \|u_N^j\|^2 \le E^*, \quad \forall n \ge 0, \end{aligned} \tag{2.7}$$

$$k \sum_{n=1}^{\infty} \|D_x^3 u_N^n\|^2 \le C_3^*, \tag{2.8}$$

where the same as Lemma 2*, the constants E^* and C_3^* are all independent of N.*