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THE ELLIPTIC TYPE NODE CONFIGURATION AND INTERPOLATION IN $R^{2*1)}$

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Abstract

In this paper, we have obtained an expression of the bivariate Vandermonde determinant for the Elliptic Type Node Configuration in R^2, and discussed the possibility of the corresponding multivariate Lagrange, Hermite and Birkhoff interpolation.

Key words: Multivariate interpolation, Polynomial interpolation, Birkhoff interpolation, Node configuration.

1. Introduction

In this paper, we use the usual multivariate notation $w^j = w_1^{j_1} \cdots w_s^{j_s}$, $|j| = j_1 + \cdots + j_s$ $(j_1, \cdots, j_s \in Z_+)$ [1,2] and let P_n be the (bivariate) polynomial space of all real (bivariate) polynomials of degree at most n.

Now we introduce the concept of the Curve Type Node Configuration (CTNC):

Definition 1. Curve Type Node Configuration A (CTNCA)[3]. *Let* $L_n = (n+1)(2n+1)$. *Then carry out the following steps:*

0. *Arbitrarily select a point as node* x_1 *in* R^2*;*

1. *Draw a quadratic irreducible curve* X_1 *such that it does not go through the node* x_1 *on* R^2 (X_1 *can be an ellipse, a hyperbola or a parabola), arbitrarily select five distinct points from* X_1 *as nodes* $x_2, \cdots, x_6$*;*

$\cdots$

n. *Draw a quadratic irreducible curve* X_n *such that it does not go through the nodes that have been selected on* R^2 (X_n *can be an ellipse, a hyperbola or a parabola), arbitrarily select* $4n+1$ *distinct points from* X_n *as nodes* $x_{L_{n-1}+1}, \cdots, x_{L_n}$.

The obtained node group $\hat{X}_n = \{x_i : i = 1, \cdots, L_n\}$ *is called the Curve Type Node Configuration A (CTNCA). If every quadratic irreducible curve is an ellipse, then* $\hat{X}_n$ *can be called an Elliptic Type Node Configuration A (ETNCA).*

Let $w = (u, v)$ be the variables in R^2 and arrange the bivariate monomial sequence $\varphi_1, \varphi_2, \varphi_3, \cdots$ as the following order:

1; u, v, u^2, uv, v^2; $u^3, u^2v, uv^2, v^3, u^4, u^3v, u^2v^2, uv^3, v^4$; $\cdots$.

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The multivariate Vandermonde determinant that we will study can be formulated as follows:

$$VD_n\begin{pmatrix}\varphi_1, & \cdots, & \varphi_{L_n}\\ x_1, & \cdots, & x_{L_n}\end{pmatrix} = \det[\phi_1, \cdots, \phi_{L_n}]$$

where the column vector

$$\phi_i = [\varphi_1(x_i), \cdots, \varphi_{L_n}(x_i)]^T.$$

If a node distribution guarantees the existence and uniqueness of a Lagrange interpolant to any given data, we say that the set of nodes admits unique Lagrange interpolation. Hence, $\hat{X}_n$ admits unique Lagrange interpolation if and only if

$$VD_n\begin{pmatrix}\varphi_1, & \cdots, & \varphi_{L_n}\\ x_1, & \cdots, & x_{L_n}\end{pmatrix} \neq 0.$$

To allow coalescence of nodes along the curves $X_1, \cdots, X_n$, we consider the following definition.

Definition 2. Curve Type Node Configuration B (CTNCB). *There exist quadratic irreducible curves $X_1, \cdots, X_n$, such that*

$$x_{L_{j-1}+1}, \cdots, x_{L_j} \in X_j \backslash (X_{j+1} \cup \cdots \cup X_n)$$

for $j = 1, \cdots, n$ as in CTNCA, where

$$x_{L_{j-1}+1}, \cdots, x_{L_j} = \underbrace{y_{j1}, \cdots, y_{j1}}_{\ell_{j1}}, \cdots, \underbrace{y_{jk_j}, \cdots, y_{jk_j}}_{\ell_{jk_j}}$$

with $\ell_{j1} + \cdots + \ell_{jk_j} = L_j - L_{j-1}$, $j = 1, \cdots, n$.

Node coalescence along X_j corresponds to Hermite interpolation with derivatives $D^k_{X_j}$ ($D^0_{X_j} := I$, the identity operator). The definition of D_{X_j} will be different according to whether X_j is an ellipse, a hyperbola or a parabola. In this paper, we will give the definition of D_{X_j} when X_j is an ellipse. We denote the column vectors by

$$D^k_{X_j}\phi_i = [D^k_{X_j}\varphi_1(x_i), \cdots, D^k_{X_j}\varphi_{L_n}(x_i)]^T.$$

Hence, the generalized Vandermonde determinant corresponding to the Hermite interpolation problem on the nodes $\hat{X}_n$ satisfying CTNCB becomes:

$$\begin{aligned}HD_n&\begin{pmatrix}\varphi_1, & \cdots, & \varphi_{L_n}\\ x_1, & \cdots, & x_{L_n}\end{pmatrix}\\ &= \det\Big[\phi_1 \vdots \cdots \vdots \underbrace{\phi_{j1} \vdots D_{X_j}\phi_{j1} \vdots \cdots \vdots D^{\ell_{j1}-1}_{X_j}\phi_{j1} \vdots \cdots \vdots \phi_{jk_j} \vdots D_{X_j}\phi_{jk_j} \vdots \cdots \vdots D^{\ell_{jk_j}-1}_{X_j}\phi_{jk_j}}_{(for\ points\ on\ X_j)} \vdots \cdots\Big].\end{aligned}$$

To allow coalescence of the quadratic irreducible curves $X_1, \cdots, X_n$, we consider the following definition.

Definition 3. Curve Type Node Configuration C (CTNCC). *The set $\hat{X}_n$ consists of distinct nodes $x_1, \cdots, x_{L_n}$, and there exist curves $X_1, \cdots, X_n$ where*

$$X_1, \cdots, X_n = \underbrace{Y_1, \cdots, Y_1}_{m_1}, \cdots, \underbrace{Y_d, \cdots, Y_d}_{m_d},$$