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DISCRETIZATION OF JUMP STOCHASTIC DIFFERENTIAL EQUATIONS IN TERMS OF MULTIPLE STOCHASTIC INTEGRALS*

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Abstract

In the Stratonovich-Taylor and Stratonovich-Taylor-Hall discretization schemes for stochastic differential equations (SDEs), there appear two types of multiple stochastic integrals respectively. The present work is to approximate these multiple stochastic integrals by converting them into systems of simple SDEs and solving the systems by lower order numerical schemes. The reliability of this approach is clarified in theory and demonstrated in numerical examples. In consequence, the results are applied to the strong discretization of both continuous and jump SDEs.

Key words: Brownian motion, Poisson process, stochastic differential equation, multiple stochastic integral, strong discretization.

1. Introduction

For the strong discretization of SDEs, any numerical method which only depends on the values of Brownian paths or Poisson paths at the partition nodes cannot achieve an order higher than 0.5 in general[2,4,8]. Therefore the evaluation of multiple stochastic integrals on the intervals between nodes is a major obstacle that must be overcome. Some attempts have been made previously in different approaches to approximate multiple stochastic integrals. [2] suggests an approximation in terms of Fourier Gaussian coefficients of the Brownian bridge process. As the layer of integration increases, the treatment becomes complicated and the computation is laborious to generate a lot of independent Gaussian random variables. For 2-dimensional Brownian motions, Gaines and Lyons applied in [1] the Marsaglia rectangle-wedge-tail method to generate stochastic area Ito integrals. However this method is not easy to be extended to general cases.

[6] indicates to model multiple Ito integrals by the rectangular rule, the trapezoidal rule as well as the Fourier method with the discussion of how small the time step should be taken to ensure the necessary accuracy. Our approach is in some sense the systematization and development of Milstein's work. In section 2, we propose to treat multiple stochastic integrals as systems of SDEs which can be solved by STH

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scheme of lower order. The results will be applied to the strong discretization of the Ginzburg-Landau system driven by both Brownian motion and Poisson process.

2. The Approximation of Multiple Stochastic Integrals

Let M be the set of the empty index and all multiple indices $\alpha = (\alpha_1, \cdots, \alpha_l)$ such that $\alpha_i \in \{-1, 0, \cdots, m\}$ for $i = 1, \cdots, l$. For $\alpha = (\alpha_1, \cdots, \alpha_l)$, define $|\alpha| = l$, $(\alpha) = \#\{\alpha_i : \alpha_i = 0\}$, $\|\alpha\| = |\alpha| + (\alpha)$. For a square integrable $\mathcal{F}_t$-predictable process f_t, define the multiple Stratonovich integrals $J_{\alpha,\rho,\tau}$ recursively by

$$J_\alpha[f]_{\rho,\tau} = \begin{cases} \int_\rho^{\tau+} J_{\alpha-}[f]_{\rho,s} dN_s^{-\alpha_l} & \text{if } \alpha_l = -1 \\ \int_\rho^{\tau} J_{\alpha-}[f]_{\rho,s} ds & \text{if } \alpha_l = 0 \\ \int_\rho^{\tau} J_{\alpha-}[f]_{\rho,s} dW_s^{\alpha_l} & \text{if } \alpha_l > 0 \end{cases} \tag{2.1}$$

and agree that $J_\alpha[f]_{\rho,\tau} = f_\tau$ when α is the empty index ϕ.

Let $B_r(L_{-2},\ L_0, \cdots, L_m)$ be the set of all formal brackets of the indeterminates $L_{-2}, L_0, \cdots, L_m$. The meaning of L_j will be clarified in section 3. For $B \in B_r(L_{-2}, L_0, \cdots, L_m)$, the degree $|B|$ is defined recursively by $|B| = |B_1| + B_2|$ and $|L_i| = 1$, $i = -2, 0, \cdots, m$. Let $\mathcal{B} \subset B_r(L_{-2}, L_0, \cdots, L_m)$ be a Philip Hall basis of $\mathcal{L}(L_{-2}, L_0, \cdots, L_m)$ with a total order $\preceq$ such that L_{-2} is the first element with respect to $\preceq$. For any $B = (ad(B_1))^j(B_2) \in \mathcal{B}$ with $B_1 \neq B_2$, we define, as in [3], the stochastic integral

$$C_{B,\rho,\tau} = \int_\rho^\tau c_{B,\rho,t} \tag{2.2}$$

where $c_{B,\rho,t}$ is defined recursively by

$$c_{B,\rho,t} = \frac{1}{j!} C^j_{B_1,\rho,t} c_{B_2,\rho,t} \tag{2.3}$$

with

$$c_{L_j,\rho,t} = \begin{cases} dt, & j = 0 \\ \circ dW_t, & j \in \{1, \cdots, m\} \end{cases} \tag{2.4}$$

and

$$c_{L_{-2},\rho,t} = dN_t. \tag{2.5}$$

Define

$$V_t^j = \begin{cases} \frac{1}{(\tau-\rho)^{1/2}} N_{(1-t)\rho+t\tau}, & j = -1, \\ \frac{(1-t)\rho + t\tau}{\tau - \rho}, & j = 0, \\ \frac{1}{(\tau-\rho)^{1/2}} W^j_{(1-t)\rho+t\tau}, & j = 0, \cdots, m, \end{cases} \tag{2.6}$$