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POTENTIAL INVERSION PROBLEMS FOR COUPLED SYSTEM OF DOWNGOING AND UPCOMING ONE-WAY WAVE EQUATIONS*1)

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Abstract

By using wave splitting method the formulation of the two-dimensional potential inversion problem is set up in terms of the coupled system for downgoing and upcoming wavefields. The boundary counditions on the characteristic surface needed for solving the problem are derived by singularity analysis. Two stability theorems are given for the direct problems of the system treated as Cauchy problems in the direction of depth.

Key words: 2-D potential inversion, Wave splitting, Singularity analysis.

1. Introduction

In this paper the following potential inversion problem is considered

$$\Big[\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial z^2}+v(x,z)\Big]p(x,z,t)=0, \quad x\in R, z>0, t>0, \tag{1.1}$$

$$p(x,z,0)=\frac{\partial p}{\partial t}(x,z,0)=0, \tag{1.2}$$

$$p(x,0,t)=\delta(t), \tag{1.3}$$

$$\frac{\partial}{\partial z}p(x,0,t)=h(x,t). \tag{1.4}$$

That is, giving an impulse at the surface $z=0$, to determine the wavefield p and potential v from the impulse response h.

In one-dimensional case, by factorizing the wave operator, the wavefield can be split into upcoming and downgoing waves, so the wave equation can be easily reduced to a coupled first-order system. The direct problem and the coefficient-inversion problem can be treated as Cauchy problems in time and in the direction of depth regarded as the time-like variable[1]. These problems are well-posed because the time and space

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variables are exchangeable in one-dimensional case. The numerical solutions for wavefields and unknown coefficients in inverse problems can be obtained layer-by-layer by finite difference methods. There are two difficulties to extent this technique into multidimensional case. The first consists in the ill-posedness of the problems, both direct and inverse, treated as Cauchy problems in the direction of depth, which is non-time-like in the multi-dimensional case. Another lies in the factorization of the wave operator, because the so-called square-root operator is not a differential one.

There are some approaches to get rid of these difficulties. A.E. Yagle and P. Raadhakrishan (1992) [2] used the clipped filter for some cutoff wave number of the lateral variable in order to avoid instability. V.H. Weston (1987) [3] used the relationship between the Dirichlet and Neumann data of the wave to construct the square-root operator. The square-root operator was considered also in L. Fishman (1991) [4] as Weyl pseudo-differential operator.

In this paper we treat only the radiation part of wavefield by neglecting the evanescent wave, which vanished rapidly with increasing of depth. In this case the square-root operator can be represented as an integral of a parameter-dependent operator. Using this representation, the two-dimensional wave equation can be reduced also to a coupled system, as in one-dimensional case, in which the two main equations describe the propagation of downgoing and upcoming wavefields and their coupling. The principal parts of these equations are exactly the one-way wave equations, familiar in geophysics for migration problems. Other equations in the system, needed for determining the auxiliary functions involved in the main equations, are one-dimensional wave equations only in the lateral variable. All these equations can easily be approximated and discretized by finite difference methods

The boundary conditions on the characteristic surface needed for solving the system of equations are derived by analyzing the propagation of the singularity.

We also give two theorems on the stability of the direct problems of the coupled system treated as Cauchy problems in the direction of depth.

2. Approximation of Square-root Operator and Wavefield Splitting

The two dimensional wave operator can be factorized as follows [5]:

$$\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial z^2} = \Big(\Lambda + \frac{\partial}{\partial z}\Big)\Big(\Lambda - \frac{\partial}{\partial z}\Big),$$

where Λ is a pseudo-differential operator, so-call square-root operator, with the "symbol"

$$\lambda(k_x, \omega) = i\sqrt{\omega^2 - {k_x}^2}.$$

We treat only the radiation part of wavefield and neglect the evanescent wave, that is, we consider only the case of $\omega^2 \geq {k_x}^2$. In this case, the following formula is true

$$\frac{1}{\pi}\int_{-1}^{1}\sqrt{1-s^2}\frac{k_x^2 ds}{\omega^2 - s^2 k_x^2} = \frac{1}{\omega}(\omega - \sqrt{\omega^2 - k_x^2}).$$