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BIVARIATE RATIONAL INTERPOLANTS WITH RECTANGLE-HOLE-STRUCTURE*1)

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Abstract

Bivariate vector valued rational interpolants are established by means of Thiele-type branched continued fractions and Samelson inverse over rectangular grids with holes, characterisation theorem with topologic structure is brought in light and uniqueness theorem in some sense is obtained.

Key words: Branched continued fraction, Interpolation, Vector-grid

1. Introduction

Given a set of distinct real points $\{x_i,\ i = 0,1,2,\cdots,\ n : x_i \in \mathbf{R}\}$ and a set of complex vector data $\{\vec{v}^{(i)},\ i = 0,1,2,\cdots,\ n : \vec{v}^{(i)} \in \mathbf{C^d}\}$, Graves-Morris showed[5] that the vector valued Thiele type continued fraction

$$\vec{S}(x) = \vec{b}^{(0)} + \frac{x - x_0}{\vec{b}^{(1)}} + \frac{x - x_1}{\vec{b}^{(2)}} + \cdots + \frac{x - x_{n-1}}{\vec{b}^{(n)}}$$

can serve to interpolate the given vectors. The construction process is closely ralated to the adoption of the Samelson inverse for vectors

$$\vec{v}^{-1} = \frac{\vec{v}^*}{|\vec{v}|^2}, \tag{1.1}$$

where $\vec{v}^*$ denotes the complex conjugate of vector $\vec{v}$. It was proved that $\vec{S}(x)$ is a vector valued rational function with numerator being a d-dimensional polynomial of degree n and denominator being a polynomial of degree $2[n/2]$, here and in the sequel of this paper, $[x]$ represents the integer function.

Let points $(x_i, y_j) \in \mathbf{R^2}$ $(i = 0,1,\cdots,\ n; j = 0,1,\cdots,m)$ be given and be arranged in the following table

$$\begin{array}{cccc} (x_0,y_0) & (x_1,y_0) & \cdots & (x_n,y_0) \\ (x_0,y_1) & (x_1,y_1) & \cdots & (x_n,y_1) \\ \vdots & \vdots & \ddots & \vdots \\ (x_0,y_m) & (x_1,y_m) & \cdots & (x_n,y_m) \end{array} \tag{1.2}$$

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which we call rectangular point-grid and denote by $\Pi^{n,m}$. Suppose d-dimensional vector $\vec{v}_{ij}$ is associated with the point (x_i, y_j) in $\Pi^{n,m}$ and let these $\vec{v}_{ij}$'s be arranged as follows

$$\begin{array}{cccc} \vec{v}_{00} & \vec{v}_{10} & \cdots & \vec{v}_{n0} \\ \vec{v}_{01} & \vec{v}_{11} & \cdots & \vec{v}_{n1} \\ \vdots & \vdots & \ddots & \vdots \\ \vec{v}_{0m} & \vec{v}_{1m} & \cdots & \vec{v}_{nm} \end{array} \tag{1.3}$$

which is called vector-grid and is denoted by $\vec{V}^{n,m}$.

Definition 1.1. *A $d-$dimensional vector valued polynomial*

$$\vec{N}(x,y) = (N_1(x,y), N_2(x,y), \cdots, N_d(x,y))$$

is said to be of degree n and denoted by $\partial\vec{N} = n$ if $\partial N_i(x,y) \le n$ for $i = 1, 2, \cdots, d$ and $\partial N_j(x,y) = n$ for some $j (1 \le j \le d)$.

Definition 1.2. *Denote by H_n the collection of all bivariate polynomials with total degree not exceeding n and by $\vec{H}_n$ the collection of d dimensional bivariate vector valued polynomials of degree n, then*

$$\vec{H}_{n,m} = \{\vec{N}(x,y)/M(x,y) | \vec{N}(x,y) \in \vec{H}_n, M(x,y) \in H_m\}$$

is called the collection of bivariate vector valued rational functions of type (n/m).

Making use of Samelson inverse and inverse differences, Zhu et al constructed the following Thiele-type branched continued fraction[9]

$$\vec{R}(x,y) = \vec{s}_0(y) + \frac{x - x_0}{\vec{s}_1(y)} + \cdots + \frac{x - x_{n-1}}{\vec{s}_n(y)}, \tag{1.4}$$

where

$$\vec{s}_l(y) = \vec{b}_{l,0}(x_0, \cdots, x_l; y_0) + \frac{y - y_0}{\vec{b}_{l,1}(x_0, \cdots, x_l; y_0, y_1)} + \cdots + \frac{y - y_{m-1}}{\vec{b}_{l,m}(x_0, \cdots, x_l; y_0, \cdots, y_m)}, \tag{1.5}$$

and $\vec{b}_{i,j}(x_0, \cdots, x_i; y_0, \cdots, y_j)$ are computed through the following recursive process

$$\vec{b}_{0,0}(x_i, y_j) = \vec{v}_{ij}, \quad i = 0, 1, \cdots, n; \ j = 0, 1, \cdots, m \tag{1.6}$$

$$\vec{b}_{0,j}(x_0; y_0, \cdots, y_j) = \frac{y_j - y_{j-1}}{\vec{b}_{0,j-1}(x_0; y_0, \cdots, y_{j-2}, y_j) - \vec{b}_{0,j-1}(x_0; y_0, \cdots, y_{j-2}, y_{j-1})} \tag{1.7}$$

$$\vec{b}_{i,0}(x_0, \cdots, x_i; y_0) = \frac{x_i - x_{i-1}}{\vec{b}_{i-1,0}(x_0, \cdots, x_{i-2}, x_i; y_0) - \vec{b}_{i-1,0}(x_0, \cdots, x_{i-2}, x_{i-1}; y_0)} \tag{1.8}$$

$$\begin{aligned} \vec{b}_{i,j}(x_0, \cdots, x_i; y_0, \cdots, y_j) = & (y_j - y_{j-1}) \Big/ [\vec{b}_{i,j-1}(x_0, \cdots, x_i; y_0, \cdots, y_{j-2}, y_j) \\ & - \vec{b}_{i,j-1}(x_0, \cdots, x_i; y_0, \cdots, y_{j-2}, y_{j-1})] \end{aligned} \tag{1.9}$$

It was shown in [9] that $\vec{R}(x,y) \in \vec{H}_{nm+n+m, 2[(nm+n+m)/2]}$ and

$$\vec{R}(x_i, y_j) = \vec{v}_{ij}, \quad i = 0, 1, \cdots, n; \ j = 0, 1, \cdots, m.$$