Journal of Computational Mathematics, Vol.17, No.2, 1999, 125–132.

THE SCHWARZ CHAOTIC RELAXATION METHOD WITH INEXACT SOLVERS ON THE SUBDOMAINS*

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Abstract

In this paper, a S-CR method with inexact solvers on the subdomains is presented, and then its convergence property is proved under very general conditions. This result is important because it guarantees the effectiveness of the Schwarz alternating method when executed on message-passing distributed memory multiprocessor system.

Key words: S-CR method, Chaotic algorithm, Inexact solvers.

1. Introduction

Early in 1869, A.H. Schwarz introduced the technique of domain decomposition and alternative iteration to prove the existence of the solution for some elliptic problem in non-regular domain. In recent years, with the arrival and tremendous development of supercomputer and multiprocessor system, this ancient and profound idea brings about fresh vitality, becomes an important source to the research of large-scale scientific computation.

Besides the ease of parallelization, Schwarz alternating algorithm and many other domain decomposition methods allow one to treat complex geometries or to isolate singular parts of the domain through adaptive refinement. They have attracted much attention all of the world, see e.g. [1], [8] for details. But all of these algorithms are synchronous, which will lead to great overheads in data communication, and severely damage the efficiency of parallelization in practice.

In [5], [6], Kang put forward the S-CR algorithm (Schwarz Chaotic Relaxation algorithm) which first combined the chaotic idea and schwarz relaxation alternating method. This new algorithm was carried out in some message - passing distributed memory multiprocessor system. Numerical experiments have showed its effectiveness[5,6]. In his Ph.D. Thesis, Huang[3,4] gave a rigorous proof for the convergence of the S-CR. This proof depends heavily on the norm estimates of some multiplicative operators.

In this article, the author will go on with the convergence analysis of the S-CR with inexact solvers on the subdomains. It is well known that implementation of the S-CR is mainly at the request of the solving of subproblems assigned on certain separate and interconnected processors. But exact solvers for these subproblems are impossible or

* Received June 6, 1994.

improper, in practice we have to employ the inexact solvers, e.g. Gauss-Seidel method, SSOR, PCG and other high efficient iterative methods. What influence on the global convergence does this result in? We show under much receivable conditions the S-CR algorithm with inexact solvers is also convergent. This result is important because it guarantees the effectiveness of the S-CR algorithm when executed on the message - passing distributed memory multiprocessor system.

Let $\Omega \subset R^2$ be a bounded polygonal domain, and let

$$\begin{cases} a(u,v) = (f,v), \quad f \in H^{-1}(\Omega), \qquad v \in H_0^1(\Omega), \\ u \in H_0^1(\Omega), \end{cases} \tag{1.1}$$

be the variational form of an elliptic operator defined on it. The bilinear form satisfies: For arbitrary $u, v \in H_0^1(\Omega)$,

$$\begin{cases} a(u,v) = a(v,u), \\ a(u,v) \le C_2 \|u\|_1 \ \|v\|_1, \\ a(v,v) \ge C_1 \|v\|_1^2, \end{cases} \tag{1.2}$$

where $\|.\|_1$ is the conventional Sobolev norm in $H_0^1(\Omega)$, C_1 and C_2 are two positive constants. We will borrow the finite element method to approximate (1.1).

Assume that the triangulation T_h is quasi-uniform[1], and let $V \subset H_0^1(\Omega)$ be the corresponding piecewise linear finite element space defined on it. Then we have the following discretized form of (1.1).

$$\begin{cases} a(u_h,v) = (f,v), \quad v \in V, \\ u_h \in V. \end{cases} \tag{1.3}$$

Thanks to (1.2), in what follows, we will consider V as a Hilbert space with inner product $a(\cdot,\cdot)$, its related induced norm is denoted by $\|.\|$.

Suppose Ω is divided into m subdomains $\Omega_1, \Omega_2, \cdots, \Omega_m$ which satisfy:

1. $\Omega = \sum \Omega_i$;

2. $\partial\Omega_i$ aligns with the triangulation T_h, i.e. the line of $\partial\Omega_i$ either coincides with or does not intersect the triangulation line of ∂T.

Let $V_i = H_0^1(\Omega_i) \cap V$ which can be looked upon as a subspace of V, $M^{\perp}$ denote the orthogonal complementary subspace of some subspace M, and P_M represent the orthogonal projection operator from V onto M. We assume that

$$V = \sum V_i. \tag{1.4}$$

Let $\omega \in (0,2)$ be a relaxation parameter. The S-CR introduced in [5] and [6] can be abstracted as follows: Let $u^0 \in V$ be an arbitrary guess function, the iterative sequence $\{u^k\}$ for solving (1.3) satisfies that

$$\begin{cases} u_1 - u^{k-1} \in V_{\tau(k)}, \\ a(u_1,v) = (f,v), \qquad v \in V_{\tau(k)}, \\ u^k = (1-\omega)u^{k-1} + \omega u_1, \end{cases} \tag{1.5}$$