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A NEW PERTURBATION SIMPLEX ALGORITHM FOR LINEAR PROGRAMMING*1)

Ping-qi Pan
(*Department of Applied Mathematics, Southeast University, Nanjing 210096, China*)

Abstract

In this paper, we first propose a perturbation procedure for achieving dual feasibility, which starts with any basis without introducing artificial variables. This procedure and the dual simplex method are then incorporated into a general purpose algorithm; then, a modification of it using a perturbation technique is made in order to handle highly degenerate problems efficiently. Some interesting theoretical results are presented. Nmerical results obtained are reported, which are very encouraging though still preliminary.

Key words: Linear programming, Simplex method, Perturbation, Dual feasibility.

1. Introduction

The dual simplex algorithm[1, 9] and the primal-dual simple algorithm [6] are well-known and efficient simplex variants. However, both of them need an initial dual feasible basis to get started, and therefore can not be directly applied to solving problems that do not have such an explicit basis. A number of schemes have been suggested to achieve dual feasibility [1, 5, 17, 18]. Some of them construct the dual analogues of the artificial-variable techniques, and none of them is as easy to implement computationally as the classical Phase-1 procedure of the primal simplex algorithm. As a result, either the dual or the primal-dual simplex algorithm is usually used only in some special cases in practice.

On the other hand, degeneracy is, in our view, all along a headache for simplex variants, including the dual and the primal-dual simplex algorithms. In practice, degeneracy occurs frequently and degrades their computational performance even thougth hardly leading to cycling. Consequently, various anti-degeneracy techniques of differing flavors have arised since the early days of linear programming. Dantzig (with his students) [4] and Charnce [2] first applied perturbation strategy to resolving degeneracy. Since then methods of this type have been proposed by, among others, Wolfe [19], Benichou, Gauthier, Hentges and Ribiere [3], Harris [8], and Gill, Murray, Saunders and Wright [7].

The distinguished features of our perturbation approach are as follows:

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(a) It applies perturbation to solving the problem itself as well as dealing with degeneracy, via a "partially revised" scheme in a naturally combined manner.

(b) It perturbs the right-hand side or the relative price row only (if necessary), and makes no change to either bounds of variables or pivot rules.

(c) The amount of perturbation can be large.

(d) No additional storage is needed.

(e) It starts from any initial basis without introducing artificial variables.

This paper is organized as follows. In Section 2, we first describe the perturbation procedure for generating a starting point for the dual or the primal-dual simplex algorithms, which starts with any basis without introducing artificial variables. In Section 3, this procedure and the dual simplex method are incorporated into a general two-phase algorithm, which is then modified through perturbation in order to handle not only usual but highly degenerate problems efficiently. Some interesting theorems concerning the perturbation approach are as well given. Finally, in Section 4, computational results are reported, which are very encouraging though still preliminary.

2. Achieving Dual Feasibility

Consider linear programming problem in the standard form:

$$\max\ z = cx \tag{2.1a}$$

$$\text{s.t. } Ax = b \tag{2.1b}$$

$$x \geq 0, \tag{2.1c}$$

where $m < n$, $A \in R^{m\times n}$ with rank $(A) = m$, $b \in R^m$, and c and x are row and column n-vectors, respectively.

Put linear system (2.1b) into the following tableau:

$$\begin{array}{c|c} -c & 0 \\ \hline A & b \end{array} \tag{2.2}$$

Suppose that an initial *simplex* tableau of the preceding presents:

$$\begin{array}{c|c} \bar{c} & \bar{z} \\ \hline \bar{A} & \bar{b} \end{array} \tag{2.3}$$

where $\bar{A} \in R^{m\times n}$, $\bar{b} \in R^m$, and for each $i = 1, \cdots, m$, the j_i-th column $\bar{a}_{j_i}$ of $\bar{A}$ is the identity vector with the i-th component 1. Then, x_{j_i}, $i = 1, \cdots, m$ are the related *basic set* of variables. We denote by J_B the set of indices of basic variables, and take the symbol:

$$\bar{J}_B = \{1, \cdots, n\}\backslash J_B. \tag{2.4}$$

Usually, tableau (2.3) is neither primally nor dually feasible, i.e., the row index set

$$I = \{i | \bar{b}_i < 0\} \tag{2.5}$$

and the index set

$$J = \{j | \bar{c}_j < 0,\ j \in \bar{J}_B\}, \tag{2.6}$$