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Subspace Search Method for Quadratic Programming with Box Constraints*[1)]

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Abstract

A subspace search method for solving quadratic programming with box constraints is presented in this paper. The original problem is divided into many independent subproblem at an initial point, and a search direction is obtained by solving each of the subproblem, as well as a new iterative point is determined such that the value of objective function is decreasing. The convergence of the algorithm is proved under certain assumptions, and the numerical results are also given.

Key words: Subspace search method, Quadratic programing, Matrix splitting

1. Introduction

In this paper, we consider the problem of minimizing a quadratic convex programming with box constrained variables:

$$\begin{aligned} &\mathrm{Min} f(x) \\ &\mathrm{s.t.} x \in \Omega \end{aligned} \tag{1.1}$$

where $\Omega = \{\ x \in R^n\colon\ l \le x \le u\}$, $f(x) = \dfrac{1}{2}x^T Hx + b^T x$, and H is an n by n symmetric positive definite matrix, and $b,\ l,\ u$ are given constant vectors in R^n.

This problem arises in several areas of applications, such as optimal control and disign engineering, linear least square problem with bounded variables and implementation of robust method for nonlinear programming, etc. Many successful algorithms for solving this type of large scale problem have been studied based on active set strategies. A popular approach is to use an active-set algorithm that solves a sequence of subproblems of the form

$$\mathrm{Min}\ f(x+d) \quad \mathrm{s.t.}\ d_i = 0, \quad i \in V_k \tag{1.2}$$

where V_k is the index set of active constraints, indicating the set of varaibles that would remain fixed at one of their bounds. Obviously, it is necessary to identify a candidate active set, and to solve the problem (1.2) exactly in the active set algorithm. Especially,

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obtaining the exact minimizer of (1.2) may require many conjugate gradient iterations, and adding constaints at a time to the working set may lead to an excessive number of iterations for large scale problem. In order to avoid the above disadvantages, a different type of algorithm, based on the gradient projection, and combination of the gradient projection with conjugate gradient, have been proposed by several authors. These algorithms have finite convergence if the problem is strictly convex and the solution is nondegenerate[12]. A similar algorithm combines conjugate gradient with gradient projection technique, and uses a new strategy for the decision of leaving the current face and make it possible to obtain finite convergence even for a singular Hessian and in the presence of dual degeneracy[7]. A primal-dual interior point algorithm is also used to solve large problem (1.1), and the numerical experiments have shown that the algorithm requires only a few steps and is very efficient[9].

In this paper, we present a subspace search method for solving the problem (1.1). The main steps of the algorithm are to divide the problem (1.1) into independent subproblems at an initial feasible point and solve each of these subproblems to obtain a search direction, and then to determine a new feasible iterative point such that the objective function is decreasing. The convergence of the algorithm is proved under certain assumptions. The main feature of the algorithm is that large scale problom (1.1) can be transformed into many small independent subproblems, and all the subproblems can be solved simultaneously.

This paper is organized as follows. In Section 2 we describe the algorithm. The convergence results are proved under certain assumptions and numerical results are also given in Section 3.

2. Derivation of the Algorithm

Now we consider the problem (1.1). Without loss of generality, assume that vector $x \in R^n$ can be divided into $(x_1^T, x_2^T, \cdots, x_t^T)$, and $x_i \in R^{n_i}$, and that $n_1 = n_2 = \cdots = n_t$ and $tn_i = n$. Accordingly, matrix H and vectors b, l, u can be also subdivided into $t \times t$ block submatrices $H_{ij} (H_{ij} \in R^{n_i \times n_i}, i, j = 1, 2, \cdots, t)$ and subvectors b_i, l_i, u_i $(b_i, l_i, u_i \in R^{n_i}$, $i = 1, 2, \cdots, t)$, respectively. Therefore, the objective function $f(x)$ can be rewritten as follows.

$$f(x) = \frac{1}{2} \sum_{i=1}^{t} \sum_{j=1}^{t} x_i^T H_{ij} x_j + \sum_{i=1}^{t} b_i^T x_i \tag{2.1}$$

Assume that an initial vector $\bar{x} \in \Omega$ is a strictly interior point, that is, $l < \bar{x} < u$, and that x belongs to the neighborhood of $\bar{x}$, then we have

$$x = \bar{x} + (x - \bar{x}) \tag{2.2}$$

Substituting (2.2) into (2.1), it is easy to derive that

$$f(x) = \frac{1}{2} \sum_{i=1}^{t} \bar{x}_i^T \hat{b}_i + \sum_{i=1}^{t} (x_i - \bar{x}_i)^T \bar{b}_i + \frac{1}{2} \sum_{i=1}^{t} \sum_{j=1}^{t} (x_i - \bar{x}_i)^T H_{ij} (x_j - \bar{x}_j) \tag{2.3}$$