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GENERAL DIFFERENCE SCHEMES WITH INTRINSIC PARALLELISM FOR SEMILINEAR PARABOLIC SYSTEMS OF DIVERGENCE TYPE*1)

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Abstract

In this paper the general finite difference schemes with intrinsic parallelism for the boundary value problem of the semilinear parabolic system of divergence type with bounded coefficients are constructed, and the existence and uniqueness of the difference solution for the general schemes are proved. And the convergence of the solutions of the difference schemes to the generalized solution of the original boundary value problem of the semilinear parabolic system is obtained. The multi-dimensional problems are also studied.

Key words: Difference scheme, Intrinsic parallelism, Semilinear parabolic system, Convergence.

1. Introduction

In [1] and [2] the general finite difference schemes having the intrinsic character of parallelism for the boundary value problems of the nonlinear parabolic system of general form (i.e., non-divergence type) are discussed under the assumption that there is an unique smooth solution for the original problem. In [3] and [4] the boundary value problems of the one-dimensional quasilinear parabolic system and multi-dimensional semilinear parabolic system of non-divergence type with bounded measurable coefficients are solved by the finite difference methods of general schemes with intrinsic parallelism. In these papers the general difference schemes with intrinsic parallelism are constructed by taking the difference approximations for the derivatives of second order to be in general the various linear combinations of the four kinds of difference quotients: the scheme ahead, the backward scheme, the scheme on the top cover of the grid, and the downward scheme. Since the parameters in the construction of the general difference schemes have large degree of freedom, in [5] some practical difference schemes are obtained by suitablely choicing these parameters for the nonlinear parabolic systems of non-divergence type. The time steplength for these difference

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schemes can be taken at least $8k$ times the time steplength for the fully explicit finite difference schemes (k can be any positive integer).

In this paper we solve the boundary value problems of the semilinear parabolic system of divergence type with bounded measurable coefficients by the finite difference methods of general schemes with intrinsic parallelism. The existence, uniqueness and convergence of the discrete vector solution for the general schemes with intrinsic parallelism are proved. Moreover, we can get some practical schemes with intrinsic parallelism by suitablely choicing the parameters in the general schemes. For these difference schemes, the time steplength can be taken at least $8k$ times the time steplength for the fully explicit finite difference schemes (k can be any positive integer), provided that the discontinuity of the coefficient matrix of the parabolic system does not occur at the interface of the domain decomposition. In the sections 2–6, we consider the case of one-dimensional problems. In the section 7 the multi-dimensional problems are discussed.

2. Difference Schemes with Intrinsic Parallelism

Let us now consider the boundary value problems for the semilinear parabolic systems of second order of the form

$$u_t = (A(x,t)u_x)_x + B(x,t,u)u_x + f(x,t,u) \tag{1}$$

where $u(x,t) = (u_1(x,t), \cdots, u_m(x,t))$ is the m-dimensional vector unknown function ($m \geq 1$), $u_t = \dfrac{\partial u}{\partial t}$, $u_x = \dfrac{\partial u}{\partial x}$ are the corresponding vector derivatives. The matrix $A(x,t)$ is an $m \times m$ positive definite coefficient matrix, and $B(x,t,u)$ is the $m \times m$ matrix, and $f(x,t,u)$ is the m-dimensional vector function. Let us consider in the rectangular domain $Q_T = \{0 \leq x \leq l,\ 0 \leq t \leq T\}$ with $l > 0$ and $T > 0$, the problem for the systems (1) with the boundary value condition

$$u(0,t) = u(l,t) = 0 \tag{2}$$

and the initial value condition

$$u(x,0) = \varphi(x) \tag{3}$$

where $\varphi(x)$ is a given m-dimensional vector function of variable $x \in [0,l]$.

Suppose that the following conditions are fulfilled.

(I) For any fixed $u \in R^m$, $A(x,t)$, $B(x,t,u)$ and $f(x,t,u)$ are bounded measurable functions with respect to $(x,t) \in Q_T$; for any fixed $(x,t) \in Q_T$, $B(x,t,u)$ and $f(x,t,u)$ are continuous with respect to $u \in R^m$; for any fixed $x \in [0,l]$, $A(x,t)$ is $m \times m$ symmetric matrix and is Lipschtiz continuous with respect to $t \in [0,T]$; and $|A(x,t)| \leq A_0$, where A_0 is a constant; and there are constants $A_1 > 0$, $B_0 > 0, C > 0$ such that $|A_t(x,t)| \leq A_1$, $|B(x,t,u)| \leq B_0$, $|f(x,t,u)| \leq |f(x,t,0)| + C|u|$.

(II) There is a constant $\sigma_0 > 0$, such that, for any vector $\xi \in R^m$, and for $(x,t) \in Q_T$,

$$(\xi, A(x,t)\xi) \geq \sigma_0|\xi|^2$$