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AN ITERATION METHOD FOR INCOMPRESSIBLE VISCOUS/INVISCID COUPLED PROBLEM VIA A SPECTRAL APPROXIMATION*1)

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Abstract

An efficient iteration-by-subdomain method (known as the Schwarz alternating algorithm) for incompressible viscous/inviscid coupled model is presented. Appropriate spectral collocation approximations are proposed. The convergence analysis show that the iterative algorithms converge with a rate independent of the polynomial degree used.

Key words: Coupled equations, Navier-Stokes equations, Euler equations, Clloca-tion approximation, Schwarz alternating algorithm.

1. Introduction

Domain decomposition methods are useful approximation techniques to face computational fluid dynamics problems, especially in complex physical domains and using parallel computational environments. They have been first employed in finite difference and finite element methods. In the context of spectral methods, they date from the late 1970s (see for instance [3] and the references therein). Earlier applications of the domain decomposition methods are related to split the whole domain into subdomains of simpler shape, and then to reduce the given problem to a sequence of subproblems which include generally same equations. Recently an intensive attention focuses on the study of possibility of using different type of equations within subdomains where different flow characters are observable. There has been some work, done mainly by Quarteroni and his collaborators [4, 8], on the coupling of compressible viscous and inviscid equations. The coupled problem of incompressible viscous and inviscid equations has been first considered by Xu and Maday in [11]. One of main goals of these investigations was to find correct conditions on the interface separating the viscous and inviscid subdomains. However efficient solvers are also of great importance when solving numerically the full time-dependent coupled equations. We propose in this paper an iteration-by-subdomain procedure to solve the coupled problem. The iteration algorithm, which involves the successive resolution of the two subproblems, is a variant of classical Schwarz alternating methods[9, 4, 8]. But the present algorithm uses two news techniques: first the norms of interface's function are defined via some interface

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"lifting" operators, different from the usual L^2-norm; secondly, the interface iteration functions are constructed on weak form, due to the discontinuous velocity/continuous pressure formulation in the inviscid subdomain (in fact we have not been able to prove the convergence of the iterative procedure based on strong form). We give exact convergence analysis and prove that the iterative algorithms using a spectral collocation approximation converge with a rate independent of the polynomial degree used.

We end this introduction by introducing some notations. Hereafter we use letters of boldface type to denote vectors and vector functions. $c, c_1, c_2, \cdots$ are generic positive constants independent of the discretization parameters. Let Ω to be a bounded, connected, open subset of R^2, with Lipschitz continuous boundary $\partial\Omega$ (see fig.1); Ω_- and Ω_+ are two open subsets of Ω, with $\Omega_- \cap \Omega_+ = \emptyset$, $\bar{\Omega}_- \cup \bar{\Omega}_+ = \bar{\Omega}$. Let $\Gamma_k = \partial\Omega \cap \partial\Omega_k, k = -, +; \Gamma = \partial\Omega_- \cap \partial\Omega_+ \neq \emptyset$. $\mathbf{n}_-, \mathbf{n}_+$ are the unit normals to Ω_-, Ω_+ respectively (so $\mathbf{n}_- = -\mathbf{n}_+$ on Γ). We notice by $C^0(\bar{\Omega})$ the space of continuous functions on $\bar{\Omega}$. For any integer m, we notice by $H^m(\Omega)$ the classical Hilbert Sobolev spaces, provided with the usual norm $\|\cdot\|_{m,\Omega}$, and also, with the semi-norm $|\cdot|_{m,\Omega}$. It is well known that the value on the boundary $\partial\Omega$ of all elements of $H^m(\Omega)$ can be given a meaning through a trace operator which maps linearly and continuously $H^m(\Omega)$ onto a subset of $L^2(\partial\Omega)$, denoted by $H^{m-1/2}(\partial\Omega)$, which is a Hilbert space for the quotient norm $\|\cdot\|_{m-1/2,\partial\Omega}$. We use also the space $L_0^2(\Omega)$ defined by

$$L_0^2(\Omega) = \{v \in L^2(\Omega); \int_\Omega v d\mathbf{x} = 0\}.$$

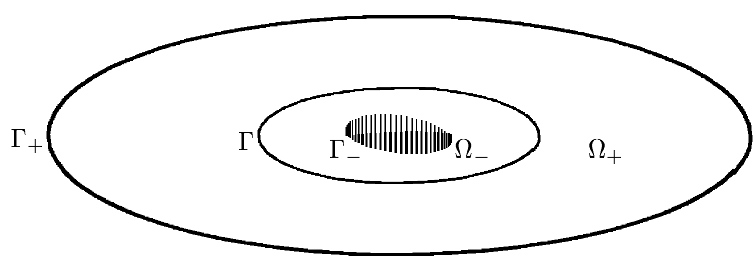

Fig.1 Computational domain

Throughout this paper, with any function $\mathbf{v}$ defined in Ω, we associate the pair $(\mathbf{v}_-, \mathbf{v}_+)$, where $\mathbf{v}_-$ (resp. $\mathbf{v}_+$) denotes the restriction of $\mathbf{v}$ to Ω_- (reps. Ω_+). We define $(\cdot,\cdot)_k, k = -, +$ and $(\cdot,\cdot)_\Gamma$ by

$$(\mathbf{u}_k, \mathbf{v}_k)_k = \int_{\Omega_k} \mathbf{u}_k \mathbf{v}_k \, d\mathbf{x}, \qquad (\Phi, \Psi)_\Gamma = \int_\Gamma \Phi\Psi \, d\sigma.$$

The scalar product on $L^2(\Omega_-)^2 \times L^2(\Omega_+)^2$,

$$(\mathbf{u}, \mathbf{v}) = (\mathbf{u}_-, \mathbf{v}_-)_- + (\mathbf{u}_+, \mathbf{v}_+)_+,$$

coincides with the usual one on $L^2(\Omega)^2$.