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A V-CYCLE MULTIGRID METHOD FOR THE PLATE BENDING PROBLEM DISCRETIZED BY NONCONFORMING FINITE ELEMENTS*

Xue-jun Xu
(*ICMSEC, Chinese Academy of Sciences, Beijing 100080, China*)
Li-kang Li
(*Department of Mathematics, Fudan University, Shanghai 200433, China*)

Abstract

In this paper, an optimal V-cycle multigrid algorithm for some famous nonconforming plate elements is established.

Key words: The plate problem, V-cycle multigrid, Nonconforming elements.

1. Introduction

Multigrid methods have become some of the most powerful methods for solving partial differential equations discretized by the finite element and finite difference methods. (cf. [7][11][14] and reference therein). Multigrid methods for the nonconforming finite elements have been studied by some reseachers recently. For the second order problems, some optimal multigrid methods for the P1 nonconforming element and the Wilson nonconforming element have been established.(cf.[5][18][22]). Mutilgrid methods for biharmonic problem have also attracted many reseachers attention, in [9][12][17], the authors presented some optimal order multigrid methods for the Morely element, but only considered W-cycle multigrid. In [21], Zhang proposed a V-cycle multigrid for Bonger-Fox-Schmit (BFS) conforming plate element, the convergence of the method rests on the nestness of the mesh spaces. But until now effictive V-cycle multigrids for the nonconforming plate elements have not been constructed.

The purpose of this paper is to develop an optimal and effective V-cycle multigrid method for some well-known nonconforming finite elements such as the Morley element, the Adini elemet. The basic idea is that no matter how finite element spaces we deal with, we insist on using the Powell-Sabin (PS) finite element space as correction space on the level l ($l = 1, ..., L-1$). The V-cycle multigrid method for the nonconforming plate elements needs smooth enough steps on the last level L, but on the coarse mesh l ($l = 1, ..., L-1$) only needs smooth one step. Moreover, because we use the PS finite element as coarse mesh spaces($l = 1..., L-1$), the intergrid transfer operator only choose the most simple interpolation opertor, the computation become very cheap.

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2. Plate Bending Problem and Nonconforming Elements

Let Ω be a convex polygonal domain in R^2, the variational form of the plate bending problem is defined as follows: Find $u \in H_0^2(\Omega)$(cf. [10] for Sobolev space notations) such that

$$a(u,v) = (f,v), \quad \forall v \in H_0^2(\Omega), \tag{2.1}$$

where f is a function on $L^2(\Omega)$ and

$$a(u,v) = \int_\Omega \triangle u \triangle v + (1-\sigma)(2\frac{\partial^2 u}{\partial x_1 \partial x_2}\frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2}\frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2}\frac{\partial^2 v}{\partial x_1^2})dx,$$

$$(f,v) = \int_\Omega fv dx,$$

and $0 < \sigma < \frac{1}{2}$ is the Possion ratio. It is well-known that (2.1) has a unique solution $u \in H_0^2(\Omega)$, and

$$a(u,v) \le C|u|_2|v|_2, \quad \forall u,v \in H_0^2(\Omega), \tag{2.2}$$

$$a(v,v) \ge C|v|_2^2, \quad \forall v \in H_0^2(\Omega), \tag{2.3}$$

where $|\cdot|_2$ is seminorm over space $H^2(\Omega)$.

Throughout this paper, c, C always denote strictly positive constant independent of h and L.

We assume the following elliptic regularity for the problem (2.1). For any $f \in H^{-1}(\Omega) = (H_0^1(\Omega))'$, there exists a solution $u \in H^3(\Omega) \cap H_0^2(\Omega)$ and

$$\|u\|_3 \le C\|f\|_{-1}.$$

It was proved in [2] that the above assumption is true if Ω is a convex polygonal domain.

We assume that Γ_h is a quasiuniform triangular or rectangular partition of Ω, let $V_h \subset L^2(\Omega)$ be a finite element space with respect to Γ_h. Define

$$a_h(u,v) = \sum_{K\in\Gamma_h}\int_K (\triangle u \triangle v + (1-\sigma)(2\frac{\partial^2 u}{\partial x_1 \partial x_2}\frac{\partial^2 v}{\partial x_1 \partial x_2} - \frac{\partial^2 u}{\partial x_1^2}\frac{\partial^2 v}{\partial x_2^2} - \frac{\partial^2 u}{\partial x_2^2}\frac{\partial^2 v}{\partial x_1^2}))dx,$$

and

$$|v|_{i,h}^2 = \sum_{K\in\Gamma_h}|v|_{i,K}^2, \quad (i = 0,1,2).$$

We assume that the above definitions satisfy:

(H1) (1). $a_h(u,v) \le C|u|_{2,h}|v|_{2,h}, \quad \forall u,v \in V_h$,

(2). $a_h(v,v) \ge C|v|_{2,h}^2, \forall v \in V_h$,

(3). $|u|_{2,h}$ *is a norm over* V_h.

(4). $D_h(u,v) \le Ch|u|_3|v|_{2,h}$, $\forall u \in H^3(\Omega)$, $v \in V_h$, *and*

$$D_h(u,v) = \sum_K \int_{\partial K}((-\triangle u + (1-\sigma)\frac{\partial^2 u}{\partial \tau^2})\frac{\partial v}{\partial n} - (1-\sigma)\frac{\partial^2 u}{\partial n \partial \tau}\frac{\partial v}{\partial \tau})ds$$

where τ *and* n *denote the unit tangential and outward normal vector along* ∂K.