Journal of Computational Mathematics, Vol.17, No.5, 1999, 545–552.

CALCULATION OF PENALTIES IN ALGORITHM OF MIXED INTEGER PROGRAMMING SOLVING WITH REVISED DUAL SIMPLEX METHOD FOR BOUNDED VARIABLES*1)

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Abstract

The branch-and-bound method with the revised dual simplex for bounded variables is very effective in solving relatively large-size integer linear programming problems. This paper, based on the general forms of the penalties by Beale and Small and the stronger penalties by Tomlin, describes the modifications of these penalties used for the method of bounded variables. The same examples from Petersen are taken and the satisfactory results are shown in comparison with those obtained by Tomlin.

Key words: Penalties, Stronger penalties, The revised dual simplex method for bounded variables.

1. Introduction

The studies on the branch-and-bound algorithm of integer programming have been carried out since 60's. The efforts in improving the algorithm are mainly concentrated on speeding up the related LP solution for each node and making better selection of node and branch for examing in order to approach the optimal solution as quick as possible. As a better strategy to estimate the problem bound and to select branch, Beals and Small proposed the penalties in 1965[2], and then Tomlin made modifications or extensions in 1969 by criterion for abandoning unprofitable branches. This stronger criterion is obtained by making use of Gomory cutting-plane constraints. These modifications have been incorporated into the famous UMPIRT system and are used successfully to solve many large practical mixed integer programming problems[3,7]. Moreover in aspect of speeding up solution of the LP problem for each node selected, the author would recommend the revised dual simplex method for bounded variables in which a branch is treated by introducing an additive bounded restriction as that with a lower or upper bound change. Thus a procedure of sensitivity analysis to this change is carried out in a continuous way based on the present basis inverse. The method works

* Received October 8, 1997.
1) This work was supported by the Chinese Postdoctoral Science Fundation.

very fast and becomes one of the main reasons of satisfactory solution speed. Since the penalties deduced by Beale and Small and th estronger penalties by Tomlin are all general formulas used for the dual simplex or revised dual simplex method without consideration of bounded variables. As further modifications or extensions, this paper describes calculation of penalties and stronger penalties for the branch-and-bound algorithm of mixed integer linear programming solving with the revised dual simplex method for bounded variables. In manifesting the effectiveness of the algorithm for bounded variable not only. But also the penalties and stronger penalties deduced by the author, the same examples from Petersen[5] are taken and the results are compared with those obtained by Tomolin.

2. Branch-and-Bound Algorithm with Revised Dual Simplex Method for Boundd Variables

The general mixed integer linear programming model with bounded variables can be put in matrix and vector forms as follows:

$$\text{Minimize } Z = CX$$

Subject to

$$\begin{aligned} &AX = b \\ &L \leq X \leq U \\ &X_k \text{ integer } K \in I \end{aligned} \tag{1}$$

Where I is the notation set of integer variables which are placed first and followed by the other continuous variables as vector elements in X.

Deducing the lower boundes as zeros by transforming $X' = X - L$ and using the same notations in (1), the problem becomes as follows:

$$\text{Minimize } Z = CX$$

Subject to

$$\begin{aligned} &AX = b \\ &0 \leq X \leq U \\ &X_k \text{ integer } K \in I \end{aligned} \tag{2}$$

For the problem at some selected node, let A be decomposed into $[B, N_1, N_2]$, where B is basis, and N_1 and N_2 consist of nonbasic columns corresponding to nonbasic variables at their lower bounds $X_{N1} = 0$ and upper bounds $X_{N2} = U$ respectively. Accordingly let R_1 being the notation set of nonbasic variables at their lower bounds, and R_2, the notation set of nobasic variables at their upper bounds. Thus the basic variables X_B and the related objective function value Z can be expressed as follows: