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ON THE CONVERGENCE OF ASYNCHRONOUS NESTED MATRIX MULTISPLITTING METHODS FOR LINEAR SYSTEMS*[1)]

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Abstract

A class of asynchronous nested matrix multisplitting methods for solving large-scale systems of linear equations are proposed, and their convergence characterizations are studied in detail when the coefficient matrices of the linear systems are monotone matrices and H-matrices, respectively.

Key words: Solution of linear systems, Asynchronous parallel iteration, Matrix multisplitting, Relaxation method, Convergence.

1. Introduction

There has been a lot of literature (see [1]–[6] and [12]) on the parallel iterative methods for the large-scale system of linear equations

$$Ax = b, \quad A \in L(R^n) \ \text{ nonsingular}, \quad x, b \in R^n \tag{1.1}$$

in the sense of matrix multisplitting since the pioneering work of O'Leary and White (see [1]) was published in 1985. One of the most recent result may be the studies on a class of asynchronous parallel matrix multisplitting relaxation methods proposed in [6]. These methods, just as was pointed out in [6], are suitable to the high speed multiprocessor systems (MIMD-systems). However, the method given in the paper requires each processor of the MIMD-system to solve a sub-system of linear equations at every iterative step. The computations of the solutions of these α sub-systems of linear equations then turn to the main tasks in concrete implementations of this

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asynchronous parallel matrix multisplitting relaxation method. Therefore, it deserves further investigation on both the method model and the convergence theory.

In this paper, through combining each iteration distributed on the corresponding processor with an inner iteration, which is used to solve its sub-system of linear equations, we construct a class of new asynchronous matrix multisplitting methods, which are called, following the customary, asynchronous nested matrix multisplitting methods. The convergence properties of these new methods are discussed in detail when the coefficient matrix $A \in L(R^n)$ is a monotone matrix as well as an H-matrix. This work can be thought of a further development of [6], and also a generalization of [9]–[10] to asynchronous matrix multisplitting methods.

For the convenience of the subsequent discussions, in the remainder of this section, we will restate the first asynchronous parallel matrix multisplitting method in [6].

We recall that a collection of triples (M_i, N_i, E_i) $(i = 1, 2, \cdots, \alpha)$ $(\alpha \leq n$, a given positive integer) is called a multisplitting of a matrix $A \in L(R^n)$ if $M_i, N_i, E_i \in L(R^n)$ $(i = 1, 2, \cdots, \alpha)$ with each E_i being nonnegatively diagonal, and satisfy: (1) $A = M_i - N_i (i = 1, 2, \cdots, \alpha)$; (2) $\det(M_i) \neq 0 (i = 1, 2, \cdots, \alpha)$; and (3) $\sum\limits_i E_i = I (I \in L(R^n)$ is the identity matrix).

Here, we have assumed that the MIMD-system considered is made up of α CPU's. Correspondingly, the following notations are also indespensable: (i) for $\forall p \in N_0 = \{0, 1, 2, \cdots\}$, $J = \{J(p)\}_{p \in N_0}$ is used to denote a sequence of nonempty subset of the set $\{1, 2, \cdots, \alpha\}$; (ii) $S = \{s_1(p), s_2(p), \cdots, s_\alpha(p)\}_{p \in N_0}$ are α infinite sequences. The sets J and S have the following properties: (a) for $\forall i \in \{1, 2, \cdots, \alpha\}$, the set $\{p \in N_0 | i \in J(p)\}$ is infinite; (b) for $\forall i \in \{1, 2, \cdots, \alpha\}, \forall p \in N_0$, it holds that $s_i(p) \leq p$; and (c) for $\forall i \in \{1, 2, \cdots, \alpha\}$, it holds that $\lim\limits_{p \to \infty} s_i(p) = \infty$.

With these preparations, the asynchronous parallel matrix multisplitting method in [6] can be described as follows:

ALGORITHM (see [6]): Suppose that we have got approximations $x^0, x^1, \cdots, x^p$ to the solution x^* of (1.1). Then the $(p+1)$-th approximation x^{p+1} of x^* can be calculated by

$$x^{p+1} = \sum_i E_i x^{i,p} \tag{1.2}$$

with $x^{i,p}$ being either x^p for $i \notin J(p)$ or the solution of the sub-system of linear equations

$$M_i x^{i,p} = N_i x^{s_i(p)} + b \tag{1.3}$$

for $i \in J(p)$.

2. Asynchronous Nested Matrix Multisplitting Methods

For the purpose of establishing our new methods, we first introduce the following concept: A collection $(M_i : F_i, G_i; N_i; E_i)$ $(i = 1, 2, \cdots, \alpha)$ is called a two-level multisplitting of a matrix $A \in L(R^n)$ if (M_i, N_i, E_i) $(i = 1, 2, \cdots, \alpha)$ is a multisplitting of it and $M_i = F_i - G_i, \det(F_i) \neq 0 (i = 1, 2, \cdots, \alpha)$. Based on this concept, by solving