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REGULARIZATION OF SINGULAR SYSTEMS BY OUTPUT FEEDBACK*1)

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Abstract

Problem of regularization of a singular system by derivative and proportional output feedback is studied. Necessary and sufficient conditions are obtained under which a singular system can be regularized into a closed-loop system that is regular and of index at most one. The reduced form is given that can easily explore the system properties as well as the feedback to be determined. The main results of the present paper are based on orthogonal transformations. Therefore, they can be implemented by numerically stable ways.

Key words: Regularization, Singular systems, Output feedback.

1. Introduction

In this paper, we consider a linear time-invariant system

$$\begin{cases} E\dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{1}$$

with the matrices $E, A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, and E is singular. The behavior of a singular system depends critically on the eigenstructure of the pencil (E, A). The pencil (E, A) and the corresponding system (1) are said to be regular if

$$\det(\alpha E - \beta A) \neq 0 \quad \text{for some } (\alpha, \beta) \in C^2 \tag{2}$$

and they are said to have index at most one if the dimension of the largest nilpotent block (which corresponds to an infinite pole) in the Kronecker canonical form of the pencil (E, A) is less than or equal to one[3,12].

A regular system of index at most one can be tansformed and seperated into a purely dynamical and a algebraic part. The algebraic part can be eliminated to give a standard system of (possibly) reduced order. Higher index singular systems can't be reduced to standard systems, and impulses can arise in the response if the control is not sufficiently smooth. The system can even lose causality[6].

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For a state feedback regularization of a singular system ($C = I$), numerous studies have been done. However, as we know, only [20, 21] investigated the output feedback regularization of singular systems and presented two numerical procedures for the feedback matrices. The results in [20, 21] can not be used straightly to study the output feedback eigenvalue assignment of the singular system because of that the reduced forms they used, for regularization of a singular system did not characterized.

The main objective of this study is to regularize a singular system by output feedback. We present some necessary and sufficient conditions for the existence of feedback matrices F, G such that the closed loop system is regular, of index at most one. The reduced form we use is a generalized controllable and observable canonical form of singular systems, so our results could form the basis of researching the eigenvalue assignment of a singular system by proportional and derivative output feedback.

This paper is arranged as folllows. Section 2 reviews some basic results and describes a reduced form relating to singular systems. Section 3 and Section 4 present and prove, respectively, necessary and sufficient conditions for output feedback regularization problem of a singular system based on the reduced form of Section 2.

2. Preliminaries

In this paper, we denote

$$r_e = \text{rank}\,(E),\ r_{eb} = \text{rank}\,[E\ B],\ r_{ec} = \text{rank}\ \begin{bmatrix} E \\ C \end{bmatrix},\ r_{ebc} = \text{rank}\ \begin{bmatrix} E & B \\ C & 0 \end{bmatrix}$$
$$S_{ebc} = \{r | r \ \text{ is an integer with } r_{eb} + r_{ec} - r_{ebc} \le r \le \min(r_{eb}, r_{ec})\}$$

We also use the following concepts.

Definition 2.1. *Let $E, A \in R^{n\times n}$, $B \in R^{n\times m}$, $[E, A]$ is regular, System (1) is completely controllable (C-controllable) if and only if*

$C0 : \text{rank}\ [\alpha E - \beta A\ B] = n, \quad \forall(\alpha, \beta) \in C^2 \backslash \{0, 0\}.$

Definition 2.2. *Let $E, A \in R^{n\times n}$, $B \in R^{n\times m}$, $[E, A]$ is regular. System (1) is strongly controlable (S-controlable) if and only if*

$C_1 : \text{rank}\ [\lambda E - A\ B] = n,\ \forall \lambda \in C;$

$C_2 : \text{rank}\ [E\ AS_E\ B] = n,$ *where the columns of S_E span $N(E)$, the nullspace of E.*

Definition 2.3. *Let $E, A \in R^{n\times n}$, $C \in R^{p\times n}$, $[E, A]$ be regular. System (1) is C-observable, S-observable if and only if $[E^T, A^T, C^T]$ satisfies conditions $C0, C1$ and $C2$, respectively.*

Obviously, if system (1) is C-controllable (C-observable), it is S-controllable (S-observable) too.

2.1 Two Basic Lemmas

We cite two well-known results. The first one will serve as a handy tool to determine if matrix pencil $[E, A]$ is regular and is index at most one. The second one indicates the allowable range of the rank of matrix $E + BGC$.