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MAXIMUM NORM ERROR ESTIMATES OF CROUZEIX-RAVIART NONCONFORMING FINITE ELEMENT APPROXIMATION OF NAVIER-STOKES PROBLEM*

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Abstract

This paper deals with Crouzeix-Raviart nonconforming finite element approximation of Navier-Stokes equation in a plane bounded domain, by using the so-called velocity-pressure mixed formulation. The quasi-optimal maximum norm error estimates of the velocity and its first derivatives and of the pressure are derived for nonconforming C-R scheme of stationary Navier-Stokes problem. The analysis is based on the weighted inf-sup condition and the technique of weighted Sobolev norm. By the way, the optimal L^2-error estimate for nonconforming finite element approximation is obtained.

Key words: Navier-Stokes problem, P1 nonconforming element, Maximum Norm.

1. Introduction

There are many research works on finite element approximation of Navier-Stokes problem in the case of lower Reynold number, by using the so-called velocity-pressure mixed formulations,e.g.[12,15,16,17,23,26]. Various sorts of conforming finite element schemes(the discrete space of velocity belongs to C^0) and nonconforming finite element schemes(the discrete space of velocity does not belong to C^0) have been discussing. It is the main reasons that the discrete spaces of velocity and pressure can not be chosen independently, they must satisfy discrete inf-sup condition (i.e. LBB condition)[1,3]. Thus, to construct conforming finite element scheme usually need some techniques for satisfying the compatibility between the discrete space of velocity and pressure and for obtaining the optimal energy norm error estimates of both velocity and pressure. Therefore, simple nonconforming finite element schemes have received considerable attention from both a theoretical and applied point of view. Nonconforming finite element scheme for stationary Stokes problem was first studied by Crouzeix and Raviart[3](this scheme is called C-R scheme). In the paper, the compatibility between the nonconforming piecewise linear triangle element and piecewise constant was shown, and the optimal energy norm error estimates of both velocity and pressure were obtained. Moreover,this scheme(i.e. C-R scheme) was extended to stationary Navier-Stokes problem

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by Teman [26], the optimal results were also obtained. The other nonconforming finite element schemes for Navie-Stokes problem may be found in [4,8,9,14,15,23,26]. But so far, maximum norm error estimates for any nonconforming finite element schemes were not considered.

Recently, the quasi-optimal maximum norm error estimates of the velocity and its first derivatives and of the pressure for conforming finite element schemes of stationary Stokes problem were studied by Duran, Nechotto and Wang[6] and Duran and Nechotto[5]. Their analyses were based on the techniques of regularized Green's functions and weighted inf-sup condition respectively. But these analyses relied on the continuity of discrete space of velocity, and did not include any nonconforming finite element schemes. In addition, L^∞ error estimates (i.e. maximum norm error estimates) for C-R scheme of nonstationary Navier-Stokes problem were consided by Rannacher[23], but its results were not quasi-optimal. The main aim of this paper is to study maximum error estimates of the velocity and its first derivatives and of the pressure for C-R scheme of stationary Navier-Stokes problem. The quasi-optimal maximum norm error estimate results for stationary problem are showen with the similar technique of weighted Sobolev norms introduced by Duran, Nechotto[5] and Rannnacher[8]. By the way, the optimal lower-norm (i.e. L^2-norm) error estimate is also shown.

The plan of this paper is the following. In section 2 we state the notations of Navier-Stokes problem and its nonconforming finite element approximations. Section 3 contains some discussion of the nonconforming finite spaces and their properties. The optimal L^2-error estimate for noncnforming finite elemet scheme is proved in section 4. In section 5 we introduce weighted Sobolev norm and give a weighted priori estimates for the solution of Stokes problem using basic solutions, and present several useful lemmas. Section 6 deals with the L^∞-error estimates for Navier-Stokes problem.

2. Notations and Prelimiaries

Let Ω be a convex polygonal domain in R^2. We consider the stationary Navier-Stokes problem for incompressible flows:

$$\begin{cases} -\gamma\triangle U + U\cdot\nabla U + \nabla p = f, & (\Omega) \\ divU = 0, & (\Omega) \\ U = 0, & (\partial\Omega) \end{cases} \tag{2.1}$$

where γ denotes the constant inverse Reynolds number. $U = (u_1, u_2)$ represents the velocity of the fluid, p its pressure and $f = (f_1, f_2)$ a diven external force. In order to write problem (2.1) in a weak form we introduce the notations:

$$X = (H_0^1(\Omega))^2, \quad M = L_0^2(\Omega), \quad L_0^2(\Omega) = \{q | q \in L^2(\Omega), \int_\Omega q dx = 0\},$$

$$a(U,V) = \gamma \int_\Omega \nabla U \cdot \nabla V dx = \gamma(\nabla U, \nabla V),$$

$$\tilde{b}(U,V,W) = \int_\Omega U\cdot\nabla V\cdot W dx = \sum_{i,j}\int_\Omega u_j\partial_j v_i w_i dx.$$