Journal of Computational Mathematics, Vol.18, No.2, 2000, 173–182.

MOMENT GENERATING FUNCTIONS OF RANDOM VARIABLES AND ASYMPTOTIC BEHAVIOUR FOR GENERALIZED FELLER OPERATORS*[1)]

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Abstract

After giving the representation of moment generating function for the S-λ type random variable by solving a differential equation, we prove that this type random variable is of regular n-r order moment. Furthermore we establish the higher order asymptotic formula for generalized Feller operators by making use of the generalized Taylor formula.

Key words: Generalized Feller operator, Moment generating function, Higher order asymptotic formula, Regular n-r order moment, Generalized Taylor formula.

1. Introduction and Notations

The generalized Feller operators which include many famous operators, such as Bernstein, Szasz-Mirakjan, Baskakov, Meyer-König and Zeller operators, can be constructed by making use of the probabilistic method. In the paper [1][2], Xu Jihua provided a general scheme for its construction, and Zhao Jinghui showed that the Feller type operators are of good approximations for unbounded functions.

Our purpose is to present representation of moment generating functions of the sum of S-λ type random variable sequence $\{\xi_i\}$ and to prove that $\{\xi_i\}$ is of the regular n-r order moment. The importance of the regular moments for studying the approximation of the probability type operators is due to the fact that the asymptotic constants are heavily dependent on them. This can be seen from the discussion in [4]. From our results the asymptotic constants of generalized Feller operators will be deduced. Furthermore we establish the higher asymptotic formula for these operators.

We briefly recall some basic concepts and elementary facts concerning probability and sketch the constructive process of the probability type operators for the paper more self-contained and completeness, refer to [1][2] for further details.

Suppose $S(x)$ is a nonnegative function which can be expanded as a power series with nonnegative coefficients p_k and convergent radius R_1:

$$S(x) = \sum_{k=0}^{\infty} p_k x^k. \tag{1}$$

* Received March 11, 1997.
[1)] Supported partially by the Natural Science Foundation of Hubei Province.

Let $\lambda(x)$ be an increasing monotone indefinitely differentiable function from $[0, R_1)$ into $[0, R)$ with $\lambda(0) = 0$. We call $S(x)$ and $\lambda(x)$ the original function of derivation and extending factor respectively. Here we note R_1 and R may be finite or infinite.

Let $\{\xi_i\}$ be an independent identically distributed random sequence. If its lattice probability

$$P(\xi_i = k) = \frac{p_k(\lambda(x))^k}{S(\lambda(x))}, \qquad k = 0, 1, ...,$$

where $p_k > 0,\ k = 0, 1, ..,$ satisfy (1), we call $\{\xi_i\}$ the S-λ type random variables.

Let $\eta_n = \sum_{i=1}^{n} \xi_i$. Its distribution function is denoted by $F_{n,x}(t)$. Let $\theta(u) := uS'(u)/S(u)$, $\varphi(x) := \theta(\lambda(x))$, and let $\sigma^2(x) := \lambda(x)\theta'(\lambda(x))$. The inverse of function $\varphi(x)$ is denoted by $\psi(x)$ whenever it exists.

The generalized Feller operators which deduced by original function of derivation $S(x)$ and extending factor $\lambda(x)$ are of the following form

$$L_n(f, x) = E\left(f(\psi(\tfrac{\eta_n}{n}))\right) = \int_0^{\infty} f(\psi(\tfrac{t}{n}))\, dF_{n,x}(t). \tag{2}$$

These operators are briefly called PPA operators in [1]. They also have the following series representation

$$L_n(f, x) = \frac{1}{(S(\lambda(x)))^n} \sum_{k=0}^{\infty} f(\psi(\tfrac{k}{n}))p_k^{(n)}(\lambda(x))^k, \quad x \in (0, R). \tag{3}$$

Here R may be finite or infinite, $p_k^{(n)}$ satisfy the recurrence formula

$$p_k^{(n)} = \sum_{j=0}^{k} p_j^{(n-1)} p_{k-j}^{(1)}, \qquad \text{where } p_{k-j}^{(1)} = p_{k-j}.$$

The paper is organized as follows. In next section we represent moment generating function for S-λ type random variable by solving a differential equation. It is important to describe the regular n-r order moment for the random variable, which is done in section 3. Finally in section 4 applying the generalized Taylor formula, we give the higher order asymptotic formula for PPA type operators and present two examples.

2. Moment generating functions of S-λ type random variables

Concerning the numerical character for S-λ type random variables $\{\xi_i\}$, we have (see [1])

$$E\xi_i = \varphi(x), \ \ D\xi_i = \sigma^2(x),$$

where E denotes the expectation of ξ_i, and D the variance. For $\eta_n = \sum_{i=1}^{n} \xi_i$, $E\eta_n = n\varphi(x)$, $D\eta_n = n\sigma^2(x)$. In order to calculate the moment generating functions of the random variable η_n, we establish the following lemma.

Lemma 1. *Let $\{L_n\}$ be the PPA type operators, defined by (2) and (3), deduced by S-λ type random variable. Then*

$$L_n(e^{\alpha\varphi(t)}, x) = \frac{[S(\lambda(x)e^{\alpha/n})]^n}{[S(\lambda(x))]^n},$$