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A GLOBALLY DERIVATIVE-FREE DESCENT METHOD FOR NONLINEAR COMPLEMENTARITY PROBLEMS*1)

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Abstract

Based on a class of functions, which generalize the squared Fischer-Burmeister NCP function and have many desirable properties as the latter function has, we reformulate nonlinear complementarity problem (NCP for short) as an equivalent unconstrained optimization problem, for which we propose a derivative-free descent method in monotone case. We show its global convergence under some mild conditions. If F, the function involved in NCP, is R_0-function, the optimization problem has bounded level sets. A local property of the merit function is discussed. Finally, we report some numerical results.

Keywords: Complementarity problem, NCP-function, unconstrained minimization method, derivative-free descent method.

1. Introduction

Consider the nonlinear complementarity problem (NCP for short), which is to find an $x \in \Re^n$ such that

$$x \geq 0, \quad F(x) \geq 0, \quad x^T F(x) = 0, \tag{1}$$

where $F : \Re^n \to \Re^n$ and the inequalities are taken componentwise. This problem have many important applications in various fields. [13, 7, 5].

Due to the less storage in computation, derivative-free descent method, which means the search direction used does not involve the Jacobian matrix of F, is popular in finding solutions of nonlinear complementarity problems. We briefly view some (not all) progress in such setting. In 1992, Fukushima [3] reformulates variational inequality problem, which includes NCP as its special element, into a constrained minimization problem through regularized gap function

$$f(x) = \max_{y \geq 0} \{(x-y)^T F(x) - \frac{1}{2\alpha}||x-y||^2\}, \qquad \alpha > 0.$$

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and propose a descent method for monotone case with global convergence. In 1993, Mangasarian and Solodov [12] reformulate NCP as an equivalent unconstrained minimization problem through the implicit Lagrangian function

$$\Psi(x) = \sum_{i=1}^{n} \psi_i(x), \tag{2}$$

where $\psi_i(x) = \phi_{MS}(x_i, F_i(x))$ and $\phi_{MS} : \Re^2 \to \Re^1$ is defined by

$$\phi_{MS}(a,b) = ab + \frac{1}{2\alpha}\left(\max^2(0, a-\alpha b) - a^2 + \max^2(0, b-\alpha a) - b^2\right), \qquad \alpha > 1.$$

Yamashita and Fukushima [19] propose a descent method for such reformulation with strong monotonicity and show the global convergence. Geiger and Kanzow [4] consider another kind of function (2) with $\psi_i(x) = \phi_{FB}^2(x_i, F_i(x))$, where $\phi_{FB} : \Re^2 \to \Re^1$ is Fischer-Burmeister NCP function defined by

$$\phi_{FB}(a,b) = \sqrt{a^2+b^2} - (a+b).$$

Also a descent method is described in [4] for monotone case. Recently, Luo and Tseng [10] study a new class of merit functions Ψ defined by

$$\Psi(x) = \psi_0(x^T F(x)) + \sum_{i=1}^{n} \psi_i(-F_i(x), x_i),$$

where $\psi_0 : \Re 1 \to [0,\infty)$ and $\psi_1, \cdots, \psi_n : \Re^2 \to [0,\infty)$ are continuous functions that are zero on the nonpositive orthant only. For such merit functions, descent methods are proposed for monotone case.

We reconsider the function ϕ_{MS}, the two parts in brackets, i.e, $(\max^2\{a-\alpha b, 0\} - a^2)$ and $(\max^2\{b-\alpha a, 0\} - b^2)$, can be viewed as relative errors with respect to a and b and the penalty parameter α. ϕ_{MS} is sum of the both parts with the third one. We note that the quantity $(\sqrt{a^2+\alpha b^2} - a)$ can be viewed as a relative error connected to $(\sqrt{a^2+b^2} - a)$ and α. $(\sqrt{\beta a^2+b^2} - b)$ can be viewed in similar way. We consider the product of the both parts.

$$\phi(a,b) = (\sqrt{a^2+\alpha b^2} - a)(\sqrt{\beta a^2 + b^2} - b), \qquad \alpha > 0, \quad \beta > 0.$$

The function ϕ is originally proposed by Peng in [15] and its elementary properties are discussed therein. It is interesting to note that ϕ is a generalization of ϕ_{FB}^2, since for $\alpha = \beta = 1$, we have

$$\phi(a,b) = (\sqrt{a^2+\alpha b^2} - a)(\sqrt{\beta a^2+b^2} - b) = \frac{1}{2}(\sqrt{a^2+b^2} - a - b)^2 = \frac{1}{2}\phi_{FB}^2.$$

Moreover, if $\alpha\beta = 1$, and let $c = \sqrt{\alpha} b$, we have

$$\begin{aligned} \phi(a,b) &= \frac{1}{\sqrt{\alpha}}(\sqrt{a^2+c^2} - a)(\sqrt{a^2+c^2} - c) \\ &= \frac{1}{\sqrt{\alpha}}\phi_{FB}^2(a,b). \end{aligned}$$