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MULTISTEP DISCRETIZATION OF INDEX 3 DAES[*1)]

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Abstract

In the past Index-3 DAEs were solved by BDF methods as multistep methods or implicit Runge-Kutta methods as one-step methods. But if the equations are nonstiff, not only BDF but other multistep methods may be applied. This paper considers four different types of multistep discretization of index 3 DAEs in hessenberg form. The convergence of these methods is proven under the condition that the multistep formula is striculy infinite stable. numerical tests also confirm the results.

Key words: Multistep methods, Adams method, BDF, DAEs, Index 3

1. Introduction

In this paper, we will consider the multistep discrezations of the differential-algebraic equations (DAEs) in Hessenberg form

$$y' = F(y,z), \tag{1.1a}$$
$$z' = K(y,z,u), \tag{1.1b}$$
$$0 = G(y), \tag{1.1c}$$

where $F \in \mathbb{R}^{N+M} \rightarrow \mathbb{R}^N$, $K \in \mathbb{R}^{N+M+L} \rightarrow \mathbb{R}^M$, $G \in \mathbb{R}^N \rightarrow R^L$, the initial value (y_0, z_0, u_0) at x_0 are assumed to be consistent, i.e., they satisfy

$$0 = G(y_0), \tag{1.2a}$$
$$0 = (G_y F)(y_0, z_0), \tag{1.2b}$$
$$0 = (G_{yy}(F,F) + G_y F_y F + G_y F_z K)(y_0, z_0, u_0). \tag{1.2c}$$

We supposes, F, G and K are sufficiently differentiable, and that

$$\|[(G_y F_z K_u)(y,z,u)]^{-1}\| \leq C. \tag{1.3}$$

in a neighbourhood of the solution. Such problems often appear in the simulation of mechanical systems with constraints and the singularly perturbed problems (see [2,

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5, 7]). BDF-methods were the first formula for solving DAEs. Convergence of BDF-methods for (1.1) was given in [3]. During the same period, the implicit Runge-Kutta (IRK) methods for solving index 3 DAEs were also considered. In [5] convergence results of IRK-methods were given, which have been sharpened by L. Jay in [8, 9]. These methods are directly derived from those methods for solving stiff ODEs. But if the equations are nonstiff, much more methods can be applied. For example, the Adams_methods, Simposon formula, etc. Then a natural question is: do these formula converge? So we need to consider the general multistep discretizations of DAEs of index 3. In [1], the multistep discretization for DAEs of index one and two were discussed. Since the numerical solution of index 3 DAEs is such more complicate than that of index 2 DAEs, different discretization may be applied to different part of the equations (1.1). There are two different types of discretization: implicit formula and explicit formula, which can be applied to (1.1a) and (1.1b) independently. So we distinguish four types of discretizations as following:

Type I: Implicit-Implicit type

$$y_{n+k} = \sum_{i=0}^{k-1} \alpha_i y_{n+i} + h \sum_{i=0}^{k} \beta_i F(y_{n+i}, z_{n+i}), \tag{1.4a}$$

$$z_{n+k} = \sum_{i=0}^{k-1} a_i z_{n+i} + h \sum_{i=0}^{k} b_i K(y_{n+i}, z_{n+i}, u_{n+i}), \tag{1.4b}$$

$$0 = G(y_{n+k}), \tag{1.4c}$$

where $y_{n+k}, z_{n+k}, u_{n+k}$ are unknown.

Type II: Implicit-Explicit type

$$y_{n+k} = \sum_{i=0}^{k-1} \alpha_i y_{n+i} + h \sum_{i=0}^{k} \beta_i F(y_{n+i}, z_{n+i}), \tag{1.5a}$$

$$z_{n+k} = \sum_{i=0}^{k-1} a_i z_{n+i} + h \sum_{i=0}^{k-1} b_i K(y_{n+i}, z_{n+i}, u_{n+i}), \tag{1.5b}$$

$$0 = G(y_{n+k}), \tag{1.5c}$$

where $y_{n+k}, z_{n+k}, u_{n+k-1}$ are unknown.

Type III: Explicit-Implicit type

$$y_{n+k+1} = \sum_{i=0}^{k-1} \alpha_i y_{n+i+1} + h \sum_{i=0}^{k-1} \beta_i F(y_{n+i+1}, z_{n+i+1}), \tag{1.6a}$$

$$z_{n+k} = \sum_{i=0}^{k-1} a_i z_{n+i} + h \sum_{i=0}^{k} b_i K(y_{n+i}, z_{n+i}, u_{n+i}), \tag{1.6b}$$

$$0 = G(y_{n+k}), \tag{1.6c}$$

where $y_{n+k+1}, z_{n+k}, u_{n+k}$ are unknown.