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SOLVING INTEGRAL EQUATIONS WITH LOGARITHMIC KERNEL BY USING PERIODIC QUASI-WAVELET[*1)]

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Abstract

In solving integral equations with logarithmic kernel which arises from the boundary integral equation reformulation of some boundary value problems for the two dimensional Helmholtz equation, we combine the Galerkin method with Beylkin's ([2]) approach, series of dense and nonsymmetric matrices may appear if we use traditional method. By appealing the so-called periodic quasi-wavelet (PQW in abbr.) ([5]), some of these matrices become diagonal, therefore we can find a algorithm with only $O(K(m)^2)$ arithmetic operations where m is the highest level. The Galerkin approximation has a polynomial rate of convergence.

Key words: Periodic Quasi-Wavelet, Integral equation, Multiscale.

1. Introduction

We try to solve the following integral equation

$$u(x) = \int_0^{2\pi} u(y)(a_0 \log |2\sin \frac{x-y}{2}| + b(x,y))dy = g(x), x \in [0, 2\pi] \qquad (1.1)$$

where a_0 is a constant, and $b(x,y)$ is a continuous function of (x,y) and is 2π periodic in each variable, which appears in exterior boundary value problems for the two-dimensional Helmholtz equation (see [9], [13], [14], [12], [24]). We want to solve the equation by using wavelets. The most important method on solving integral equations was introduced in [3], but the method introduced in [3] can not be applied directly

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to this equation. Recently, Beylkin and Brewster introduced a new method called Multiscale Strategy in [2]. But when we apply Beylkin's ([2]) method, there appears dense and nonsymmetric matrices, which leads to large complexity. We therefore appeal to the so-called PQW, some of the matrices become diagonal.

Our idea of construction of PQW traces back to the sources of multiresolution analysis and the orthogonal periodic spline functions (see [15]). In [15] the author constructed periodic orthonormal splines (the scaling function), but they did not give the wavelets. Koh, Lee and Tan ([11]) and Tasche [22] constructed periodic wavelets by using Fourier coefficients of some functions and by using some special techniques. For instance, in [22], the author, constructed the periodic wavelets by appealing the Euler Frobenius function. Our construction is manipulating the periodic B-spline directly, and the construction of mother wavelets is very simple such that the decomposition and reconstruction formulas for the coefficient involve only two non-zero terms, that behave as the Haar basis. By using this wavelet, the complexity in solving the integral equation is much smaller. Since our wavelet has no localization, we call it quasi-wavelet [5].

In recent years, this equation and it's numerical solution have received much attention in the literature. A considerable part of the research on the numerical solution of this integral equation is concerned with the application of Galerkin methods, collocation methods and qualocation methods and their error analysis. For details, we refer the reader to [13], [23], [10] and the references therein.

Wavelets, which are originally developed for signal and image processing ([7]), has been applied in solving partial differential equations([1], [8]) and integral equations ([3], [18], [19], [21]). The latest paper that we received was written by Chen, Micchelli and Xu (see [6]) which deal with second kind of integral equations with singular kernel by using multiwavelet. In this paper, we introduce periodic quasi-wavelets and its application to solving equation (1.1). Here, FFT and Multiscale Strategy introduced in [2] are the key techniques. Multiresolution analysis (MRA) was introduced by Meyer [17] and Mallat [16] as a general framework for construction of the wavelet bases. Using MRA, the notion of the non–standard representation of operators was introduced in