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GLOBAL SUPERCONVERGENCES OF THE DOMAIN DECOMPOSITION METHODS WITH NONMATCHING GRIDS[*1)]

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Abstract

In this paper, the global superconvergences of the domain decomposition methods with Lagrangian multiplier and nonmatching grids are proven for solving the second order elliptic boundary value problems. Moreover, L^{∞} and L^2 error estimates are discussed and a defect correction scheme is presented.

Key words: Domain decomposition, Defect correction, Global superconvergence, Nonmatching grids.

1. Introduction

A nonconforming domain decomposition method with Lagrangian multipliers was proposed in [13]. The basic idea of this method is to deal with the nonconforming of nonmatching grids by introducing the Lagrangian multipliers on interfaces of subdomains and its advantages are that it allows not only the incompatibility of the internal variables on the interface between subdomains, but also the discontinuity of the boundary variables on the common vertices of subdomains. Thus one can choose different mesh size, interpolating function and type of element in different subdomains according to the different requirement of pratical problems. So this method is very flexible and well suitable to parallel computational environment.

The superconvergence estimates and error expansions for the finite element method are well studied in many papers. We refer to Chen [5], Křížek and Neittanmäki [11], Lin and Xu [15], Lin and Zhu [14,27], Křížek [12] and Wahlbin [25] for a detail and survey and to Lin and Zhu [14] for some techniques of high accuracy analysis.

Even so, there still remains some foundmental problems to need studying. In particularly, for high accuracy analysis of the domain decomposition with nonmatching grids, it has seldom been found in the literature. This paper just studies this problem and gives its global superconvergence estimates and defect correction.

2. Domain Decomposition and Global Superconvergence

Let our problem be to solve the following differential equation

$$\begin{cases} -\triangle u + u &= f \quad \text{in} \quad \Omega \\ \qquad\quad u &= 0 \quad \text{on} \quad \partial\Omega \end{cases} \tag{1}$$

where Ω is a convex smooth domain and $f \in L^2(\Omega)$.

Throughout the paper, Ω is assumed a rectangle to simplify the discussion, although the results are valid for convex smooth domain (cf [14]). We first shall divide the domain Ω into rectangular subdomains Ω_j $(j = 1, \dots, n_d)$ and then subdivide subdomains Ω_j into rectangular meshes T_h^j with the two widths h_j and k_j, where $h = \max\{h_j, k_j\}$. Let A_j (according to certain

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order) be the vertices of $\partial\Omega_j$, d the diameter of Ω_j and Γ_{ij} the straight edge of $\partial\Omega_j$ with the vertices A_{j-1} and A_j. Let $\partial\Omega_j = \bigcup_i \Gamma_{ij}$ and $\Gamma = \bigcup_{i,j}\Gamma_{ij}$.

Now let us define the functional spaces

$$H(\Omega) = \prod_{j=1}^{n_d} H^1(\Omega_j) \text{ and } H(\Gamma) = \prod_{j=1}^{n_d} H^{-\frac{1}{2}}(\partial\Omega_j)$$

with the norm

$$||v||_\Omega^2 = \sum_{j=1}^{n_d} ||v||_{1,\Omega_j}^2 \text{ and } ||\mu||_\Gamma^2 = \sum_{j=1}^{n_d} ||\mu||_{-\frac{1}{2},\partial\Omega_j}^2$$

respectively, where

$$||u||_{\frac{1}{2},\partial\Omega_j} = (d^{-1}||u||_{0,\partial\Omega_j}^2 + |u|_{\frac{1}{2},\partial\Omega_j}^2)^{\frac{1}{2}},\ |u|_{\frac{1}{2},\partial\Omega_j}^2 = \int_{\partial\Omega_j}\int_{\partial\Omega_j} \frac{(u(x)-u(y))^2}{(x-y)^2}dxdy$$

and

$$||u||_{-\frac{1}{2},\partial\Omega_j} = \sup_{\substack{v\in H^{\frac{1}{2}}(\partial\Omega_j)\\ |v|\neq 0}} \frac{|\int_{\partial\Omega_j} uv|}{||v||_{\frac{1}{2},\partial\Omega_j}}.$$

Then $(v,\mu) \in \mathbf{H}$ if and only if $v \in H(\Omega)$ and $\mu \in H(\Gamma)$, where $\mathbf{H} = H(\Omega)\times H(\Gamma)$ with the norm $||(v,\mu)||_{\mathbf{H}}^2 = ||v||_\Omega^2 + ||\mu||_\Gamma^2$. Obviously, $\mathbf{H}$ is a Hilbert space.

We may introduce a bilinear form

$$B(u,\lambda;v,\mu) = \sum_{j=1}^{n_d}\{\int_{\Omega_j}(\sum_{i=1}^{2}\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i} + uv)dx - \int_{\partial\Omega_j}(v\lambda + u\mu)ds\},\ \ \forall(u,\lambda),(v,\mu)\in\mathbf{H},$$

and a functional

$$F(v,\mu) = \sum_{j=1}^{n_d}\int_{\Omega_j} fvdx.$$

Then the weak form of problem (1) is defined as follows

$$\begin{cases} \text{find} \quad (u,\lambda) & \in \mathbf{H} \text{ such that} \\ B(u,\lambda;v,\mu) & = F(v,\mu),\ \ \forall(v,\mu)\in\mathbf{H}. \end{cases} \tag{2}$$

We know by [13] that the problem (2) has one and only one solution $(u_0,\lambda_0)\in\mathbf{H}$. Let $w\in H^1(\Omega)$ be the weak solution of problem (1). Then $u_0 = w$ and $\lambda_0 = \frac{\partial w}{\partial\vec{n}}$, where $\vec{n}$ is the unit outer normal of Ω_j.

For any positive integer k,l,n,m_j, we define the finite element spaces as follows:

$$S_h(\Omega_j) = \{v\in C(\Omega_j)|\, v|_e \in Q_{m_j}(e), \forall e\in T_h^j\}$$

and

$$S_n(\partial\Omega_j) = \{\mu|\,\mu|_{\Gamma_{ij}} \in P_n(\Gamma_{ij}), \forall\Gamma_{ij}\subset\Gamma\},$$

where

$$Q_{m_j}(e) = \text{span}\{x^k y^l : \quad 0\le k,l\le m_j\}$$