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ATTRACTORS FOR DISCRETIZATION OF GINZBURG-LANDAU-BBM EQUATIONS*[1)]

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Abstract

In this paper, Ginzburg-Landau equation coupled with BBM equation with periodic initial boundary value conditions are discreted by the finite difference method in spatial direction. Existence of the attractors for the spatially discreted Ginzburg-Landau-BBM equations is proved. For each mesh size, there exist attractors for the discretized system. Moreover, finite Hausdorff and fractal dimensions of the discrete attractors are obtained and the bounds are independent of the mesh sizes.

Key words: Attractor, Spatially discreted, Ginzburg-Landau-BBM equations, Hausdorff and fractal dimensions.

1. Introduction

In this paper, we consider the following periodic initial value problem for the system of Ginzburg-Landau equation coupled with BBM equation

$$\varepsilon_t + \nu\varepsilon - (\alpha_1 + i\alpha_2)\varepsilon_{xx} + (\beta_1 + i\beta_2)|\varepsilon|^2\varepsilon - i\delta n\varepsilon = g_1(x), \tag{1.1}$$

$$n_t + nn_x + \gamma n - \alpha n_{xx} - n_{xxt} + |\varepsilon|_x^2 = g_2(x), \tag{1.2}$$

$$\varepsilon(x+2\pi, t) = \varepsilon(x,t), \quad n(x+2\pi,t) = n(x,t), \tag{1.3}$$

$$\varepsilon(x,0) = \varepsilon_0(x), \quad n(x,0) = n_0(x). \tag{1.4}$$

where $\varepsilon(x,t)$ is a complex function, $n(x,t)$ is a real scalar function, ν, α, δ, γ, α_1, α_2, β_1, β_2 are real constants, and $g_1(x), g_2(x)$ are given real functions.

This problem describes the nonlinear interactions between Langmuir wave and ion acoustic wave in plasma physics, $\varepsilon(x,t)$ denotes electric field, $n(x,t)$ the perturbation of density (see [1, 9, 2]). In [3] Guo proved the global existence of smooth solution ($\alpha_1 = \nu = 0$), and in [6], Guo and Jiang considered the periodic initial value problem with the weak dissipative case and obtained the upper bounds of Hausdorff and fractal dimensions for the global attractors, both are on a bounded domain. In [7] Guo and Jiang studied the existence of the global attractors of the problem (1.1)-(1.2) and (1.4) on an unbounded domain.

Although the existence and uniqueness of the global smooth solution for the problem (1.1)-(1.4) in one dimension have been obtained, but we still do not know if the existence of the attractors for the spatially finite difference equations of the problem (1.1)-(1.4) is available and the attractors independent of the mesh sizes. Moreover, we still need to know if the finite

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dimensionality of the attractors still hold for the spatially finite difference equations. On the other hand, in order to simulate the properties of the solution numerically, we need to discrete the equations. These are the main questions we would like to consider. In this paper, we discrete Ginzburg-Landau equation coupled with BBM equation with the periodic initial value conditions by the finite difference method in spatial directions. It is proved that for each mesh size, there exist attractors for the discretized systems. The bounds for the Hausdorff and fractal dimensions of the discrete attractors are obtained, and the various bounds are independent of the mesh size.

2. Discretization of Ginzburg-Landau-BBM Equations and a Priori Estimates

Let $\Omega = [0, 2\pi]$ be a domain in real one dimensional Euclidean space $\mathbf{R}$, and J a nonnegative integer and $h = \frac{2\pi}{J}$ the space step length. The interval Ω is divided into discrete lattice $\Omega_h = \{x_1, x_2, \cdots, x_J\}$, where $x_j = jh$. The discretized function $u = (u_1, u_2, \cdots, u_J)^T$ and $u_j = u(x_j)$.

The difference operators are defined by

$$u_{jx} = \frac{1}{h}(u_{j+1} - u_j) = \nabla_h u_j, \quad u_{j\overline{x}} = \frac{1}{h}(u_j - u_{j-1}),$$

$$u_{j\hat{x}} = \frac{1}{2h}(u_{j+1} - u_{j-1}) = \frac{1}{2h}\Delta_0 u_j, \;\; u_{jx\overline{x}} = \frac{1}{h^2}(u_{j+1} - 2u_j + u_{j-1}) = \Delta_h u_j.$$

Spatially finite difference discretized version of the problem (1.1)-(1.4) can be defined by

$$\frac{d}{dt}\varepsilon_j + \nu\varepsilon_j - (\alpha_1 + i\alpha_2)\varepsilon_{jx\overline{x}} + (\beta_1 + i\beta_2)|\varepsilon_j|^2\varepsilon_j - i\delta n_j\varepsilon_j = g_{1j}, \tag{2.1}$$

$$\frac{d}{dt}n_j + D_0 f(n_j) + \gamma n_j - \alpha n_{jx\overline{x}} - n_{jx\overline{x}t} + |\varepsilon_j|^2_{\hat{x}} = g_{2j}, \tag{2.2}$$

$$\varepsilon_{j+rJ}(t) = \varepsilon_j(t), \quad n_{j+rJ}(t) = n_j(t), \tag{2.3}$$

$$\varepsilon_j(0) = \varepsilon_0(x_j), \quad n_j(0) = n_0(x_j). \tag{2.4}$$

where $D_0 f(n_j) = \frac{1}{3}n_j\Delta_0 n_j + \frac{1}{3}\Delta_0(n_j^2)$. $\nu, \gamma, \alpha, \alpha_1, \beta_1 > 0$, r is an integer, $j = 1, 2, \cdots, J$.

Denote the scalar product of two discrete complex periodic functions $u = \{u_j \,|j = 1, 2, \cdots, J\}$ and $v = \{v_j \,|j = 1, 2, \cdots, J\}$ by

$$(u, v)_h = \sum_{j=1}^{J} u_j\overline{v}_j h.$$

Here $\overline{v}_j$ denotes the complex conjugate of v_j. For the norms of the discrete function u and its difference quotients $\nabla_h^k u$ of order $k > 0$, we take the expressions

$$\|\nabla_h^k u\|_s = \left(\sum_{j=1}^{J-k} |\nabla_h^k u_j|^s h\right)^{\frac{1}{s}}, \quad 1 \le s < \infty$$

and

$$\|\nabla_h^k u\|_\infty = \max_{j=1,\cdots,J-k} |\nabla_h^k u_j|,$$

where $\nabla_h^k = \nabla_h \cdot \nabla_h \cdots \nabla_h$ and $k \ge 0$ is any nonnegative integer and p is a real number.

Now we state some interpolation relations between the norms of several difference quotients for the discrete function u on the finite interval Ω.