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MULTISCALE FINITE ELEMENT METHOD FOR SUBDIVIDED PERIODIC ELASTIC STRUCTURES OF COMPOSITE MATERIALS*1)

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Abstract

In this paper, from the view of point of macro- and meso- scale coupling, we discuss the mechanical behaviour for subdivided periodic elastic structures of composite materials. A multiscale numerical method and its error estimate are reported. Finally, numerical experiments results supports strongly the theoretical ones presented in the paper.

Key words: Multi-scale asymptotic method, Finite element method, Composite medium.

1. Introduction

For a kind of elastic structures of composite materials of which the geometric and physical parameters are of some periodicity, e.g., common laminated plate and shell, and fibre reinforced, particle reinforced, and woved composite materials and so forth , we can regard them as the periodic structures with a unit cell.

Generally speaking, it is extremely difficult to compute directly the above elastic structures by using usual FEM, due to the complicated geometric configurations and highly oscillatory physical parameters. To overcome this crucial difficulty, I.Babuska, J.L.Lions et al. [1] proposed early homogenization method. However, homogenization method can only reflect the macroscopic average properties of elastic structures, but does not describe the local mechanical behaviour. To this end, J.L.Lions and O.A.Oleinik et al. [2,5] obtained complete asymptotic expansions for the Dirichlet boundary value problems of the second order elliptic equation and the linear elastic structures of composite materials in perforated domains, respectively. Jun-zhi Cui and Li-qun Cao et al. [6,7] obtained the complete asymptotic expansions for the Dirichlet boundary value problems of the second order elliptic equation and the linear elastic system with rapidly oscillating coefficients in domains formed by entirely basic configurations, respectively. In present paper, we will propose the multiscale FEM for subdivided elastic structures of composite materials.

The organization of this paper is as follows. In section 2 , we shall obtain multiscale asymptotic expansion and truncation error estimates for subdivided elastic structures of composite materials. Section 3 is devoted to the FE computation of periodic solutions $N_\alpha(\xi)$ and the modified homogenized linear elastic system $\overline{U}^0(x)$. In section 4 , a multiscale FE scheme and total error estimates are given. Finally, numerical experiments results are reported and are coincident with the theoretical ones.

In what follows summation over repeated Latin indices from 1 to n is assumed. If the vectors u, v or matrices A, B have elements belonging to a Hilbert space $\mathcal{H}$ with a scarlar product $(\cdot,\cdot)_{\mathcal{H}}$, we use the following notations:

$$(u,v)_{\mathcal{H}} = (u_i, v_i)_{\mathcal{H}}, \quad \|u\|_{\mathcal{H}} = (u,u)_{\mathcal{H}}^{1/2}$$
$$(A,B)_{\mathcal{H}} = (a_{ij}, b_{ij})_{\mathcal{H}}, \quad \|A\|_{\mathcal{H}} = (A,A)_{\mathcal{H}}^{1/2}$$

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and write $u, v \in \mathcal{H}$; $A, B \in \mathcal{H}$ instead of $u, v \in \mathcal{H}^2$; $A, B \in \mathcal{H}^{n^2}$.

2. Multiscale Asymptotic Expansion and Truncation Error Estimates

Without loss of generality, we discuss only the elastic structure as shown in Figure 2.1. Let $\bar{\Omega} = \bar{\Omega}_1 \cup \bar{\Omega}_2$, $\Omega_1 \cap \Omega_2 = \emptyset$, $\bar{\Omega}_1 = \bigcup_{z \in T_\varepsilon} \varepsilon(z + \bar{Q})$ be the elastic composite structures formed by entirely basic configurations, $T_\varepsilon = \{z \in Z^n : \varepsilon(z+Q) \subset \Omega\}, Q = \{\xi : \quad 0 < \xi_j < 1, \quad j = 1, 2, \cdots n\}$

To begin with, introduce the following notations:

Displacement boundary Γ_u, force boundary Γ_σ, $\partial\Omega = \Gamma_u \bigcup \Gamma_\sigma$, $mes(\Gamma_u) > 0$, $\Gamma^* = \partial\Omega_1 \bigcap \partial\Omega_2$, body force $f = (f_1, \cdots f_n)^T$; boundary force $\phi(x) = (\phi_1, \cdots \phi_n)^T$; strains $\varepsilon_{ij} = \frac{1}{2}(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$; stresses σ_{ij}; constitutive relation $\sigma_{ij} = C_{ij}^{pq}(x, \frac{x}{\varepsilon})\varepsilon_{pq}$, where

$$C^{pq} = \left(C_{ij}^{pq}(x, \frac{x}{\varepsilon})\right) = \begin{cases} A^{pq} = \left(a_{ij}^{pq}(\frac{x}{\varepsilon})\right) & \text{if} \quad x \in \overline{\Omega}_1 \\ B^{pq} = \left(b_{ij}^{pq}\right) & \text{if} \quad x \in \Omega_2 \end{cases} \tag{2.1}$$

$p, q, i, j = 1, 2, \cdots n$

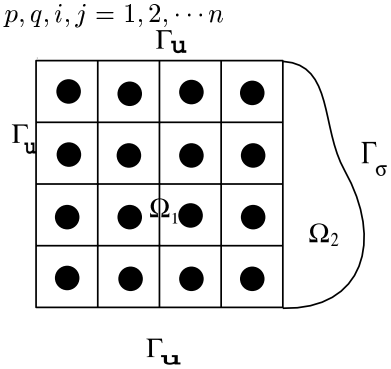

Figure 2.1

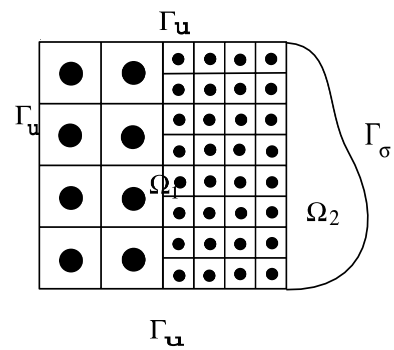

Figure 2.2

Definition 2.1. *We say that a family of matrices* $A^{pq}(\frac{x}{\varepsilon}) = A^{pq}(\xi)$, $\xi = \varepsilon^{-1}x$, $p, q = 1, 2, \cdots n$, *belongs to class* $E(\mu_1, \mu_2)$, *if their elements* $a_{ij}^{pq}(\xi)$ *are bounded measurable functions satisfying the following conditions:*

(1) $a_{ij}^{pq}(\xi)$ *are 1-periodic in* ξ

(2)

$$a_{ij}^{pq}(\xi) = a_{ji}^{qp}(\xi) = a_{pj}^{iq}(\xi); \tag{2.2}$$

(3) $\mu_1 \eta_{ip}\eta_{ip} \leq a_{ij}^{pq}(\xi)\eta_{ip}\eta_{jq} \leq \mu_2 \eta_{ip}\eta_{ip}$

where η_{ip} *is an arbitray symmetric matrix with real elements,* μ_1, μ_2*=const*> 0

Let

$$b_{ij}^{pq} = \lambda\delta_{ip}\delta_{jq} + \mu\delta_{ij}\delta_{pq} + \mu\delta_{iq}\delta_{jp} \tag{2.3}$$

where $\lambda > 0$, $\mu > 0$ are the Lame constants, δ_{ij} is the Kronecker notation.

$$\textbf{Equations of equilibrium} : -\sigma_{pq,q} = f_p, \quad p = 1, 2 \cdots n, \quad \text{in} \quad x \in \Omega, \tag{2.4}$$

$$\text{i.e.} \qquad -\mathcal{L}_\varepsilon U^\varepsilon \equiv -\frac{\partial}{\partial x_p}(C^{pq}(x, \frac{x}{\varepsilon})\frac{\partial U^\varepsilon(x)}{\partial x_q}) = f(x), \quad x \in \Omega \tag{2.5}$$

where

$$U^\varepsilon(x) = \begin{cases} u^\varepsilon(x) & x \in \overline{\Omega}_1 \\ w(x) & x \in \Omega_2 \end{cases} \tag{2.6}$$

$$\textbf{Displacement boundary condition} : \quad U^\varepsilon(x) = u^\varepsilon(x) = \bar{u}(x) \qquad x \in \Gamma_u \tag{2.7}$$

$$\textbf{Force boundary condition} : \quad \sigma_\varepsilon(U^\varepsilon) = \sigma(w) = \nu_p B^{pq}\frac{\partial w}{\partial x_q} = \phi(x) \qquad x \in \Gamma_\sigma \tag{2.8}$$

$$\textbf{Interface conditions} : \quad u^\varepsilon(x)|_{\Gamma^*} = w(x)|_{\Gamma^*}, \quad \sigma_\varepsilon(u^\varepsilon)|_{\Gamma^*} = -\sigma(w)|_{\Gamma^*} \tag{2.9}$$