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CURVILINEAR PATHS AND TRUST REGION METHODS WITH NONMONOTONIC BACK TRACKING TECHNIQUE FOR UNCONSTRAINED OPTIMIZATION*1)

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Abstract

In this paper we modify type approximate trust region methods via two curvilinear paths for unconstrained optimization. A mixed strategy using both trust region and line search techniques is adopted which switches to back tracking steps when a trial step produced by the trust region subproblem is unacceptable. We give a series of properties of both optimal path and modified gradient path. The global convergence and fast local convergence rate of the proposed algorithms are established under some reasonable conditions. A nonmonotonic criterion is used to speed up the convergence progress in some ill-conditioned cases.

Key words: Curvilinear paths, Trust region methods, Nonmonotonic technique, Unconstrained optimization.

1. Introduction

Trust region method is a well-accepted technique in nonlinear optimization to assure global convergence. One of the advantages of the model is that it does not require the objective function to be convex. Many different versions have been suggested in using trust region technique. For each iteration, suppose a current iterate point, a local quadratic model of the function and a trust region with center at the point and a certain radius are given. A point that minimizes the model function within the trust region is solved as a trial point. If the actual reduction achieved on the function f at this point is satisfactory comparing with the reduction predicted by the quadratic model, the point is accepted as a new iterate, then the trust region radius is adjusted and the procedure is repeated. Otherwise, the trust region radius should be reduced and a new trial point needs to be determined. Recently Bulteau and Vial proposed in [1] curvilinear paths with trust region method for unconstrained optimization and a main feature of which is instead of minimizing a quadratic function within the whole trust region which is a hyperball, it only minimizes the function over a simple curvilinear path inside the trust region. In other words, their method is an approximate trust region method via curvilinear paths.

It is also noticed that Nocedal and Yuan [9] suggested a combination of the trust region and line search method. The motivation is intuitive. As we know, in traditional trust region method, after solving a subproblem we need to use some criterion to check if the trial step is acceptable. If not, the subproblem must be resolved with a reduced trust region radius. It is possible that the trust region subproblem needs to be resolved many times before obtaining an acceptable solution, and hence the total computation for completing one iteration might be expensive. A plausible remedy is that at an unsuccessful trial step we switch to the line search

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technique by employing the back tracking steps. Of course the prerequisite for being able to making this shift is that although the trial step is unacceptable as next iterative point, it should provide a direction of sufficient descent.

Another valuable idea is to abandon the traditional monotonic decreasing requirement for the sequence $\{f(x_k)\}$ of the objective values (see [3] and [7]), because monotonicity may cause a series of very small steps if the contours of objective function f are a family of curves with large curvature.

The main purpose of this paper is to modify and improve the curvilinear path type approximate trust region method by adopting the above ideas: back tracking and nonmonotonic search. In particular, we shall show that the trial step generated by their subproblem produces a sufficiently descent direction. We shall focus on unconstrained optimization

$$\min_{x\in\Re^n} f(x).$$

Both theoretical analysis and numerical experiment will be undertaken for the improved algorithms.

The paper is organized as follows. In section 2, we give expressions to the curvilinear path model trust region steps and propose the characterizations and properties of the curvilinear paths quadratic subproblem. In Section 3, we describe the algorithm which combines the techniques of trust region, back tracking and nonmonotonic search. In section 4, weak global convergence of the proposed algorithm is established. Some further convergence properties such as strong global convergence and superlinear convergence rate are discussed in section 5. Finally, the results of numerical experiments are reported in section 6.

2. Curvilinear Paths

In trust region algorithms, an important portion of the unconstrained minimization procedure will be concerned with the solution of a subproblem of the form

$$\begin{aligned} \min \quad & q_k(\delta) \stackrel{\text{def}}{=} f_k + (g^k)^T\delta + \frac{1}{2}\delta^T B_k \delta \\ \text{s.t.} \quad & \|\delta\| \le \Delta_k \end{aligned} \tag{2.1}$$

where $f_k = f(x_k),\ g^k = \nabla f(x_k),\ \delta = x - x_k,\ B_k$ is either $\nabla^2 f(x_k)$ or its approximation, $q_k(\delta)$ is the local quadratic approximation of f and Δ_k is the trust region radius and $\|\cdot\|$ throughout is 2-norm. Let δ_k be the solution of the subproblem. Then set next step

$$x_{k+1} = x_k + \delta_k. \tag{2.2}$$

Based on solving the about trust region subproblem, we given the following Lemmas which are due to Sorensen paper in [13].

Lemma 2.1. *s is a solution to the subproblem (2.1) in which x_k is given by x if and only if s is a solution to the following equations of the forms*

$$(B + \mu I)\ s = -g(x) \tag{2.3}$$

$$\mu(\|s\|^2 - \Delta^2) = 0, \quad \mu \ge 0. \tag{2.4}$$

and $B + \mu I$ is positive semidefinite.

Lemma 2.1 establishes the necessary and sufficient conditions concerning the pair $\mu,\ s$ when s solves (2.1). The next results are immediate consequences of lemma 2.1.

Lemma 2.2. *Let $\mu \in \Re^1,\ s \in \Re^n$ be a solution to the following equations of the forms*

$$(B + \mu I)\ s = -g(x) \tag{2.5}$$

and $B + \mu I$ is positive semidefinite. Then we have that
(1) *if $\mu = 0$ and $\|\delta\| \le \Delta$ then s solves the subproblem (2.1);*
(2) *if $\|\delta\| = \Delta$ then s solves*

$$\min q_x(\delta) \quad \text{subject to} \quad \|\delta\| = \Delta;$$