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PARALLEL CHAOTIC MULTISPLITTING ITERATIVE METHODS FOR THE LARGE SPARSE LINEAR COMPLEMENTARITY PROBLEM*1)

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Abstract

A parallel chaotic multisplitting method for solving the large sparse linear complementarity problem is presented, and its convergence properties are discussed in detail when the system matrix is either symmetric or nonsymmetric. Moreover, some applicable relaxed variants of this parallel chaotic multisplitting method together with their convergence properties are investigated. Numerical results show that highly parallel efficiency can be achieved by these new parallel chaotic multisplitting methods.

Key words: Linear complementarity problem, Matrix multisplitting, Chaotic iteration, Relaxed method, Convergence property.

1. Introduction

We consider the linear complementarity problem LCP(M,q): Find a $z \in \mathbb{R}^n$ such that

$$Mz + q \geq 0, \qquad z \geq 0, \qquad z^T(Mz+q) = 0,$$

where $M = (m_{ij}) \in \mathbb{R}^{n \times n}$ and $q = (q_i) \in \mathbb{R}^n$ are given real matrix and vector, respectively. This problem arises in various scientific computing areas such as the Nash equilibrium point of a bimatrix game (e.g., Cottle and Dantzig[4] and Lemke[12]) and the free boundary problems of fluid mechanics (e.g., Cryer[8]). There have been a lot of researches on the approximate solution of the linear complementarity problem LCP(M,q). For details one can refer to Cottle, Pang and Stone[6] and references therein. These researches presented efficient iterative methods and systematic convergence theories for solving the linear complementarity problem in the sequential computing environment.

To solve the linear complementarity problem in the parallel computing environment, Machida, Fukushima and Ibaraki[14] recently presented a multisplitting iterative method by making use of the matrix multisplitting technique introduced in O'Leary and White[17]. Under suitable conditions about the weighting matrices and the multiple splittings, Machida, Fukushima and Ibaraki[14] and Bai[1] proved the convergence of this method for symmetric and nonsymmetric linear complementarity problems, respectively. This method possesses good parallel computational properties, and it is much suitable for implementing on the synchronous parallel multiprocessor systems. It can achieve high parallel efficiency provided the workloads among the processors of the multiprocessor system are well balanced. When such a balance can be obtained, the individual processor is then ready to contribute towards their update of the global iterate almost at the same time, which, in turn, minimizes idle time. However, such a balance

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of workload is not always available in many applications, and the mutual wait among the processors of the multiprocessor system is usually inevitable, which, hence, decreases the parallel efficiency of the multisplitting method.

To avoid loss of time and efficiency in processor utilization, in this paper, we propose a chaotic multisplitting iterative method for solving parallely the linear complementarity problem LCP(M,q). In the implementation of this method on the multiprocessor system, each processor can carry out its local iterate a varying number of steps until a mutual phase time is reached when all processors are ready to contribute towards the global iteration. Hence, the synchronous wait among different processors is greatly decreased while the efficient numerical computation on each processor is largely increased. This, therefore, makes the new chaotic multisplitting method achieve high parallel efficiency. Under the same restrictions on the weighting matrices and the multiple splittings as in [14] and [1], we establish the convergence theories of this new method for both the symmetric and nonsymmetric linear complementarity problems. Moreover, for the convenience of practical implementations, some relaxed explicit variants of the above chaotic multisplitting method are presented, and their convergence for both the symmetric and nonsymmetric linear complementarity problems are discussed in detail as well. At last, with a lot of numerical results, we show that the new chaotic multisplitting methods are feasible and efficient for parallely solving the linear complementarity problems on the multiprocessor systems.

2. Preliminaries

First of all, we briefly review some necessary notations and concepts in [1] and [14]. A matrix $A=(a_{ij})\in\mathbb{R}^{n\times n}$ is called a monotone matrix if it is nonsingular and satisfies $A^{-1}\geq 0$; an M-matrix if it is a monotone matrix and satisfies $a_{ij}\leq 0$ for $i\neq j$, $i,j=1,2,\cdots,n$; an H-matrix if its comparison matrix $\langle A\rangle$ is an M-matrix, where $\langle A\rangle=(\langle a_{ij}\rangle)\in\mathbb{R}^{n\times n}$ is defined by $\langle a_{ii}\rangle=|a_{ii}|$ for $i=1,2,\cdots,n$, and $\langle a_{ij}\rangle=-|a_{ij}|$ for $i\neq j$, $i,j=1,2,\cdots,n$; an H_+-matrix if it is an H-matrix having positive diagonal elements; and a Q-matrix if the LCP(A,b) has a solution for any $b\in\mathbb{R}^n$. A sufficient condition for $A\in\mathbb{R}^{n\times n}$ to be a Q-matrix is that either A is an H_+-matrix[1] or A is a strictly copositive matrix [6]. In the former case, the LCP(A,b) always has a unique solution for every $b\in\mathbb{R}^n$. For a given matrix $A\in\mathbb{R}^{n\times n}$, let $F,G\in\mathbb{R}^{n\times n}$ be such that $A=F+G$. Then (F,G) is called a splitting of the matrix A. The splitting (F,G) is called a convergent splitting if the spectral radius of the matrix $(F^{-1}G)$ is less than one, i.e., $\rho(F^{-1}G)<1$. It is called an M-splitting if F is an M-matrix and $G\leq 0$; an H-splitting if $\langle F\rangle-|G|$ is an M-matrix; an H-compatible splitting if $\langle A\rangle=\langle F\rangle-|G|$; and a Q-splitting if F is a Q-matrix. In particular, the splitting (F,G) is called an H_+-splitting and H_+-compatible splitting if it is an H-splitting and H-compatible splitting, respectively, with F an H_+-matrix. Let $N_0=\{0,1,2,\cdots\}$ and $\{A_p\}_{p\in N_0}$ be a sequence of matrices in $\mathbb{R}^{n\times n}$. Then we call $A_p(p\in N_0)$ positive definite uniformly in p if there exists a positive constant c, independent of p, such that $z^TA_pz\geq cz^Tz$ holds for all $z\in\mathbb{R}^n$.

The following lemmas, proved in [20] and [9] respectively, will frequently be used in the sequel.

Lemma 2.1. [20] *Let $A\in\mathbb{R}^{n\times n}$ have nonpositive off-diagonal entries. Then A is an M-matrix if and only if there exists a positive vector $u\in\mathbb{R}^n$ such that $Au\geq 0$.*

Lemma 2.2. [9] *Let $A\in\mathbb{R}^{n\times n}$ be an H-matrix, $D=diag(A)$, and $A=D-B$. Then:*

(a) *A is nonsingular;*

(b) *$|A^{-1}|\leq\langle A\rangle^{-1}$; and*

(c) *$|D|$ is nonsingular and $\rho(|D|^{-1}|B|)<1$.*

When the system matrix $M\in\mathbb{R}^{n\times n}$ is symmetric, associated with the LCP(M,q) is the following quadratic programming problem QP(M,q):