Journal of Computational Mathematics, Vol.19, No.3, 2001, 299–308.

A HOMOTOPY METHOD OF SWITCHING SOLUTION BRANCHES AT THE PITCHFORK BIFURCATION POINT*1)

Yu-hao Cong
(*College of Mathematical Sciences, Shanghai Teachers University, Shanghai 200234, China*)

Zheng-you Zhu
(*Shanghai Institute of Applied Mathematics and Mechanics, Shanghai 200072, China*)
(*Department of Mathematics, Shanghai University, Shanghai 200072, China*)

Abstract

By introducing proper parameters in the original nonlinear system, a continuation method for switching solution branches at a pitchfork point is proposed and their theories have been established in this paper. It is sufficient to implement this method that a standard continuation procedure is only used. Some numerical examples are given to illustrate the effectiveness of this method.

Key words: Homotopy method, Switching solution branches, Pitchfork point.

1. Introduction

In order to compute all solutions of a nonlinear system of equations, a numerical method is needed for changing solution branches at bifurcation points. Suppose that a bifurcation point (x^*, λ^*) hase been located. H.B.Keller [4] proposed a method of switching solution branches at (x^*, λ^*) by means of distinct roots of a homogeneously quadratic system of equations, W.C.Rheinboldt [5] gave a method for switching solution branches at a simple bifurcation point, using a singular chord method, which does not require the bifurcation point to be known accurately. The methods mentioned above depend on the asymptotic expressions of bifurcation solutions near a bifurcation point in principle. In this paper we present a new method for switching the solution branches which does not depend on the asymptotic expressions of bifurcation solutions. In our method, firstly we make an unfolding of the original system by introducing proper parameters. Then by using the standard continuation method in [1] [4] we track a solution curve of the unfolding problem to realize the switching of solution branches. Our method like the method described in [5] also does not require the bifurcation point to be known and any information of the second order derivatives. Because it is sufficient to implement our method that the standard continuation software is only used, our method is more effective and simpler than the methods mentioned above. In addition, as our method independs on the the asymptotic expressions of bifurcation solutions, one may expect that this method is valid for many kinds of bifurcation points.

In the second section, we discuss carefully the numerical method for switching solution branches for the standard pitchfork problem. Although the solutions of this problem are obtained straightforwards, yet it is necessary to understand the principle of our method and to analyse the general cases. In the third section, we discuss the case of system of equations and show the validity of the method of this paper. Finally some numerical examples are given to demonstrate the effectiveness of our method in the fourth section.

* Received December 18, 1998.
1)The project is supported by NNSF of China, the Youth Science Foundation of Shanghai Higher Education (99QA80) and Science Foundation of Shanghai Science and Technique Committee (00JC14057).

2. The Case of Standard Pitchfork

In this section, the problem of standard pitchfork:

$$f(\xi,\lambda) := \xi^3 - \lambda\xi = 0 \tag{1}$$

will be examined in detail. Obviously, the bifurcation diagram of (1),namely, the solution set of (1) consists of two curves $\Gamma_1 : \xi = 0$ and $\Gamma_2 : \xi^2 = \lambda$ which intersect at the pitchfork bifurcation point $(\xi,\lambda) = (0,0)$ (see Fig.1.)

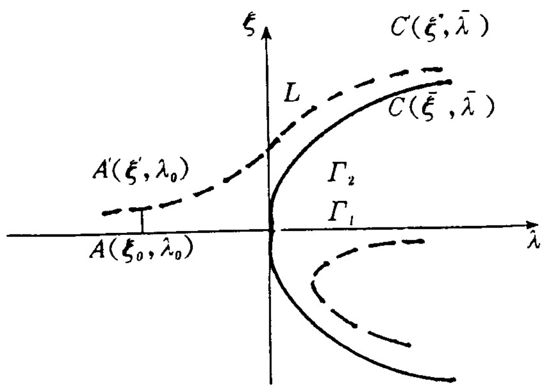

Fig.1. The bifurcation diagram of (1) and (2).

Now assume that by tracking the solution curve Γ_1, we get a point A on Γ_1 near the pitchfork point $(0,0)$, the coordinate of A is denoted by (ξ_0,λ_0),where $\lambda_0 < 0$. To obtain a point on Γ_2, we make a one-parameter unfolding of $f(\xi,\lambda)$[3] as follows:

$$g(\xi,\lambda;\beta) := \xi^3 - \lambda\xi - \beta = 0. \tag{2}$$

The bifurcation diagram of (2) with $\beta \neq 0$ is shown by the broken lines in Fig1. The original idea for realizing the switching of solution branches is represented as follows. In the first stage, we choose any ξ' near ξ_0 such that $\beta' = f(\xi',\lambda_0) \neq 0$, the point (ξ',λ_0) is denoted by A' in the Fig1. Obviously, we have $g(\xi',\lambda_0;\beta') = 0$. As 0 is the regular value of $g(\bullet,\bullet;\beta')$[2] , we track solution curve L of the equation $g(\bullet,\bullet;\beta') = 0$ passing A' by using the standard continuation method until obtain a point $C'(\xi'',\bar{\lambda})$ on L in which $\bar{\lambda} > 0$ in the second stage. As 0 is also the regular value of $g(\bullet,\bar{\lambda};\bullet)$, by using the continuation method once again, track the solution curve of the equation $g(\bullet,\bar{\lambda};\bullet) = 0$ passing point (ξ'',β') until get the solution $(\bar{\xi},0)$ in the third stage. Finally, we will show that $(\bar{\xi},\bar{\lambda})$ denoted by C in Fig.1 is just a desired point on Γ_2. According to above frame, two continuation procedures are needed for switching solution branches. However, if we allow to change the value of β' in the second stage, the three stage in above frame can be combined into one continuation procedure. The following describe the details of our method mentioned above, consider the system of equations:

$$H(\xi,\lambda;s) := \begin{pmatrix} \xi^3 - \lambda\xi - \alpha\theta(\xi,\lambda;\alpha) \\ \lambda - \lambda_0 - \eta s \end{pmatrix} = 0, \tag{3}$$

in which $\alpha = \alpha(s) := \epsilon s(1-s)$, θ is a smooth function and $\theta(0,0;0) \neq 0$,η and ϵ are given parameters. Next we shall show that if (ξ_0,λ_0) is sufficiently close to pitchfork point $(0,0)$ and