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THE NUMERICAL METHODS FOR SOLVING EULER SYSTEM OF EQUATIONS IN REPRODUCING KERNEL SPACE $H^2(R)^{*1)}$

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Abstract

A new method is presented by means of the theory of reproducing kernel space and finite difference method, to calculate Euler system of equations in this paper. The results show that the method has many advantages, such as higher precision, better stability, less amount of calculation than any other methods and the reproducing kernel function has good local properties and its derived function is wavelet function.

Key words: Euler system of equations, Reproducing kernel method, Finite difference method, Wavelet function.

1. Introduction

In recent years, more and more people are interested in solving Euler system of equations. They presented various methods to simulate the flow of the complicated fluid field. It is well known that Euler system of equations has described many practical engineering problems, such as spherically symmetric flow, the flow inside a pipe, whose sectional area changed slowly, the radius of curvature is large, sectional area is small and so on. And it not only describes the incompressible ideal fluid one-dimension unsteady flow but also is the foundation for solving Navier-Stokes system of equations, which describes the viscous flow, hence it lies at the heart of fluid mechanics, that is why, solving Euler system of equations possesses important significance. But the various methods given for solving Euler system of equations so far are only confined to classical ones, such as finite difference and finite element methods, their effects are not very well. This paper combines the reproducing kernel with the finite difference method and gives the approximate solution to Euler system of equations. Numerical experiment results indicate that the effect is very well. Because the reproducing kernel function possesses the good local properties, such as the odd order vanishing moment, symmetry and regularity, its dilation has fast attenuation etc., its derived function is a wavelet function which possesses even order vanishing moment and anti-symmetry, regularity and its dilation has fast attenuation. So reproducing kernel function possesses its operation superiority and the description superiority of wavelet function, that is why the superiority of this method prevails over others. Firstly, this method can command simply, conveniently and easily. Secondly, it can apply extensively, especially solving complicated non-linear partial difference equations, which are solved hardly. Thirdly, the method possesses the advantages of higher precision and better stability compared with the others. In addition, we can construct two-dimension reproducing kernel space by tensor product form and extend it to two-dimension Euler system of equations.

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2. Reproducing Kernel Space $H^2(R)$

2.1 Definition of reproducing kernel space $H^2(R)$

Definition. *$H^2(R) = \{u(x)|u(x)$ and $u'(x)$ are absolutely continuous function in R, u, u' and $u'' \in L^2(R)\}$. Inner product on $H^2(R)$ is defined as follows:*

$$< u, v >= \int_R (uv + 2u'v' + u''v'')dx.$$

By reference [1], we know $H^2(R)$ is a reproducing kernel space and its reproducing kernel is:

$$k_2(x-\xi) = \frac{1}{4}(1 + |x-\xi|)e^{-|x-\xi|},$$

namely

$$\forall u \in H^2(R), \quad < u(\xi), K_2(x-\xi) >= u(x).$$

Theorem. *Let $\{x_i\}_{i=1}^n$ denote a system where x_i are pairwise different nodes in R, and let $\phi_i(x) = K_2(x - x_i)$, then $\{\phi_i(x)\}_{i=1}^n$ is a linearly independent function system in $H^2(R)$, we obtain an orthonormal system $\{\phi_i^*(x)\}_{i=1}^n$ by Schmidt orthogonalization, where $\phi_i^*(x) = \sum\limits_{k=1}^{i} \alpha_{ki}\phi_k(x)$, when $\{x_i\}_{i=1}^\infty$ is dense in R, $\{\phi_i^*\}_{i=1}^\infty$ is an orthonormal basis in $H^2(R)$, namely*

$$\lim_{n\to\infty}(H_n u)(x) = u(x), \text{ where } u(x) \in H^2(R), \ (H_n u)(x) = \sum_{i=1}^{n}(u, \phi_i^*)\phi_i^*.$$

Proof. Let

$$C_1\phi_1 + C_2\phi_2 + \cdots + C_n\phi_n = 0 \tag{2.1}$$

We use proof by contradiction. Assume $\{\phi_i(x)\}_{i=1}^n$ is linearly independent, we apply Fourier transform to both sides of the (2.1), then we get

$$C_1e^{-ix_1\omega} + C_2e^{-ix_2\omega} + \cdots C_ne^{-ix_n\omega} = 0. \tag{2.2}$$

We may as well let $C_1 \neq 0$, obtain

$$1 = -(\frac{C_2}{C_1}e^{i(x_1-x_2)\omega} + \cdots + \frac{C_n}{C_1}e^{i(x_1-x_n)\omega}) \tag{2.3}$$

By (2.3), we know there is one that doesn't equal zero in $C_1, \cdots, C_n$, setting $C_2 \neq 0$, then $\frac{C_2}{C_1} \neq 0$. Let $x_1 - x_2 = y_1, \cdots, x_1 - x_n = y_n$, $\frac{C_2}{C_1} = a_2$, $\frac{C_n}{C_1} = a_n$, and by deriving (2.3), we get

$$0 = -(a_2y_2e^{-iy_2\omega} + \cdots + a_ny_ne^{-iy_n\omega}). \tag{2.4}$$

Obviously $y_2, \cdots, y_n \neq 0$, let $a_2y_2 = b_2, \cdots, a_ny_n = b_n$, then (2.4) is converted into (2.5)

$$0 = -(b_2e^{-iy_2\omega} + \cdots + b_ne^{-iy_n\omega}). \tag{2.5}$$

Since $b_2 \neq 0$, at least there is one of them doesn't equal zero in $b_3, \cdots, b_n$, setting $b_3 \neq 0$, and so on and so forth, we get $c_ne^{-iy_n\omega} = 0$ $(c_n \neq 0)$ which leads to an absurdity. So $\{\phi_i\}_{i=1}^n$ are linearly independent.