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# A TRUST REGION ALGORITHM FOR CONSTRAINED NONSMOOTH OPTIMIZATION PROBLEMS*1)

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**Abstract**

This paper presents a new inexact trust region algorithm for solving constrained nonsmooth optimization problems. Under certain conditions, we prove that the algorithm is globally convergent.

*Key words*: Trust region method, Nonsmooth function, Constrained optimization, Global convergence.

## 1. Introduction

Trust region methods are an important class of iterative methods for solving nonlinear optimization problems, and have been developed rapidly in recent twenty years (see [1]–[9], [15], [16] etc.). For nonsmooth optimization problems, as early as in 1984, Y. Yuan [2] [3] proposed a trust region method for the composite function $f(x) = h(g(x))$, where $h$ is convex and $g \in C^1$; L. Qi and J. Sun [4] proposed an inexact trust region method for the general unconstrained nonsmooth optimization problems; A. Friedlander et al. [7] also proposed an inexact trust region method for the box constrained smooth optimization problems, and J.M. Martínez and A.C. Moretti [8] extended the method to the nonsmooth case with convex polyhedron constraints.

In this paper, we will consider the nonsmooth case with the general closed convex set constraint, i.e., we will consider the following constrained nonsmooth optimizaton problem:

$$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & x \in \Omega. \end{array} \tag{1}$$

where $\Omega$ is a closed convex set in $R^n$ and $f : \Omega \to R$ is a locally Lipschitzian function in $\Omega$.

We propose a new inexact trust region algorithm for solving (1), where the subproblem is similar to that in [8], but the adjustment of the trust region radius is different. In contrast to the model in [8], our method is suitable to more general closed convex set constraint. Moreover, we discuss the convergence not only for the case $\{\|B_k\|\}$ uniformly bounded, but also for the case

$$\|B_k\| \le c_5 + c_6 \sum_{i=1}^{k} \Delta_i, \forall k \tag{2}$$

or

$$\|B_k\| \le c_7 + c_8 k, \forall k \tag{3}$$

where $c_5, c_6, c_7, c_8$ are all constants, $\Delta_i$ is the trust region radius in the i-th iteration.

The organization of the remainder of this paper is as follows. In section 2, we introduce the concept of the critical point, describe the algorithm and make some basic assumptions on the

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algorithm. In section 3, we establish the global convergence of the algorithm when $\{\|B_k\|\}$ is uniformly bounded or (3) holds.

## 2. Algorithm and Basic Assumptions

Throughout this paper, $\|\cdot\|$ denotes the 2-norm. In fact, according to the equivalence of the norms in $R^n$, the results in this paper hold for arbitrary norms.

Let $\varphi(x,s) : \Omega \times R^n \to R$ be a given iterative function satisfying the following assumptions:

**A1:** For all $x \in \Omega$, $\varphi(x,0) = 0$ and $\varphi(x,\cdot)$ is lower semicontinuous.

**A2:** For all $x \in \Omega$, if $x+s \in \Omega$, $t \in [0,1]$, then $\varphi(x,ts) \le t\varphi(x,s)$.

The concept of iterative functions can be found in [13].

**Definition 1.** *For all $x \in \Omega, \Delta > 0$ and $M \ge 0$ , let*

$$m(x,\Delta,M) = \min\{\varphi(x,s) + \frac{1}{2}M\|s\|^2 : x+s \in \Omega \text{ and } \|s\| \le \Delta\} \tag{4}$$

*we say that $x \in \Omega$ is a critical point of (1) if there exist $\Delta > 0, M \ge 0$ such that $m(x,\Delta,M) = 0$.*

The above definition on the critical point is similar to that in [8], we also may use the definition on the critical point in [4]. In fact, it is easy to see that under Assumptions A1 and A2, the two definitions are equivalent.

Next, we introduce an useful lemma whose proof can also be found in [8].

**Lemma 1.** *Under Assumptions A1 and A2, $x \in \Omega$ is a critical point of (1) if and only if for all $\Delta > 0$ and $M \ge 0$,*

$$m(x,\Delta,M) = 0.$$

Now, we assume that $x_0 \in \Omega$ is the initial point, the level set $L_0 = \{x | f(x) \le f(x_0), x \in \Omega\}$ is bounded, $D \subset R^n$ is a bounded open set containing $L_0$. Let $\Delta_0$ be the diameter of $D$, and $c_0, c_1, c_2, c_3, c_4$ be constants satisfying $0 < c_0 \le 1$, $0 < c_2 \le c_1 < 1$, $c_3 < 1 < c_4$. A trust region algorithm for solving constrained nonsmooth optimization problem (1) is stated as follows:

**Algorithm 1.**

For all $k \ge 0$

**Step 0.** Choose the symmetric matrix $B_k \in R^{n\times n}$ and constant $M_k$ satisfying $\|B_k\| \le M_k$.

**Step 1.** Solve the subproblem

$$\begin{array}{ll} \text{minimize} & Q_k(s) = \varphi(x_k,s) + \frac{1}{2}M_k\|s\|^2 \\ \text{subject to} & x_k + s \in \Omega \text{ and } \|s\| \le \Delta_k. \end{array} \tag{5}$$

Assume that the solution of (5) is $s_k^Q$.

If $Q_k(s_k^Q) = 0$, stop; otherwise

**Step 2.** Compute $s_k \in R^n$ such that

$$\Phi_k(s_k) \le c_0 Q_k(s_k^Q) \text{ and } x_k + s_k \in \Omega, \|s_k\| \le \Delta, \tag{6}$$

where, for all $s \in R^n$, $\Phi_k(s) = \varphi(x_k,s) + \frac{1}{2}s^T B_k s$.

**Step 3.** Let

$$r_k = \frac{f(x_k+s_k) - f(x_k)}{\Phi_k(s_k)}, \tag{7}$$

$$x_{k+1} = \begin{cases} x_k + s_k, & \text{if } r_k > c_2, \\ x_k, & \text{otherwise ,} \end{cases} \tag{8}$$