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NUMERICAL SOLUTION OF MULTIMEDIUM FLOW WITH VARIOUS DISCONTINUITIES*

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1. Introduction

In this paper we discuss the numerical solution of the interaction of the strong plane explosion wave with the boundary between two gases.

Because this kind of unsteady problem has a characteristic length and there are shocks, contact discontinuities and rarefaction waves, it is a good test problem for judging methods.

The singularity-separating method presented in [1] can accurately solve this kind of problem. Its scheme is unconditionally stable and possesses a second order accuracy. We have obtained satisfactory numerical results using the singularity-separating method for this complicated problem.

We shall show the singularity-separating method and its difference scheme in detail. Also we shall give the fractional errors of mass, momentum and energy for our numerical results. The error estimations show that our solutions are accurate.

Finally, we shall compare this method with the *L-W* scheme for a special problem.

2. Formulation of the Problem

We consider the following problem of plane unsteady motion of a perfect gas. The system of equations of gas dynamics is

$$\begin{cases} \dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=0, \\ \dfrac{\partial \rho}{\partial t}+u\dfrac{\partial \rho}{\partial x}+\rho\dfrac{\partial u}{\partial x}=0, \\ \dfrac{\partial p}{\partial t}+u\dfrac{\partial p}{\partial x}+\gamma p\dfrac{\partial u}{\partial x}=0. \end{cases} \tag{1}$$

Suppose that at the time $t=-t_1(t_1>0)$, the state of flow field is $\gamma=\gamma_l$ (ratio of specific heats), $\rho=\rho_l$ (density), $u=u_l=0$ (velocity), $p=p_l=0$ (pressure), $e=e_l=0$ (internal energy) at $x<0$ and $\gamma=\gamma_r$, $\rho=\rho_r$, $u=u_r=0$, $p=p_r=0$, $e=e_r=0$ at $x>0$. There is a strong explosion on the plane $x=R_0$ (see Fig. 1). "Strong" explosion means that the internal energy and the pressure of the static gas in front of the explosion wave can be neglected as compared with the energy released per unit area E. The explosion wave AO moves towards the boundary of two gases. It meets the boundary

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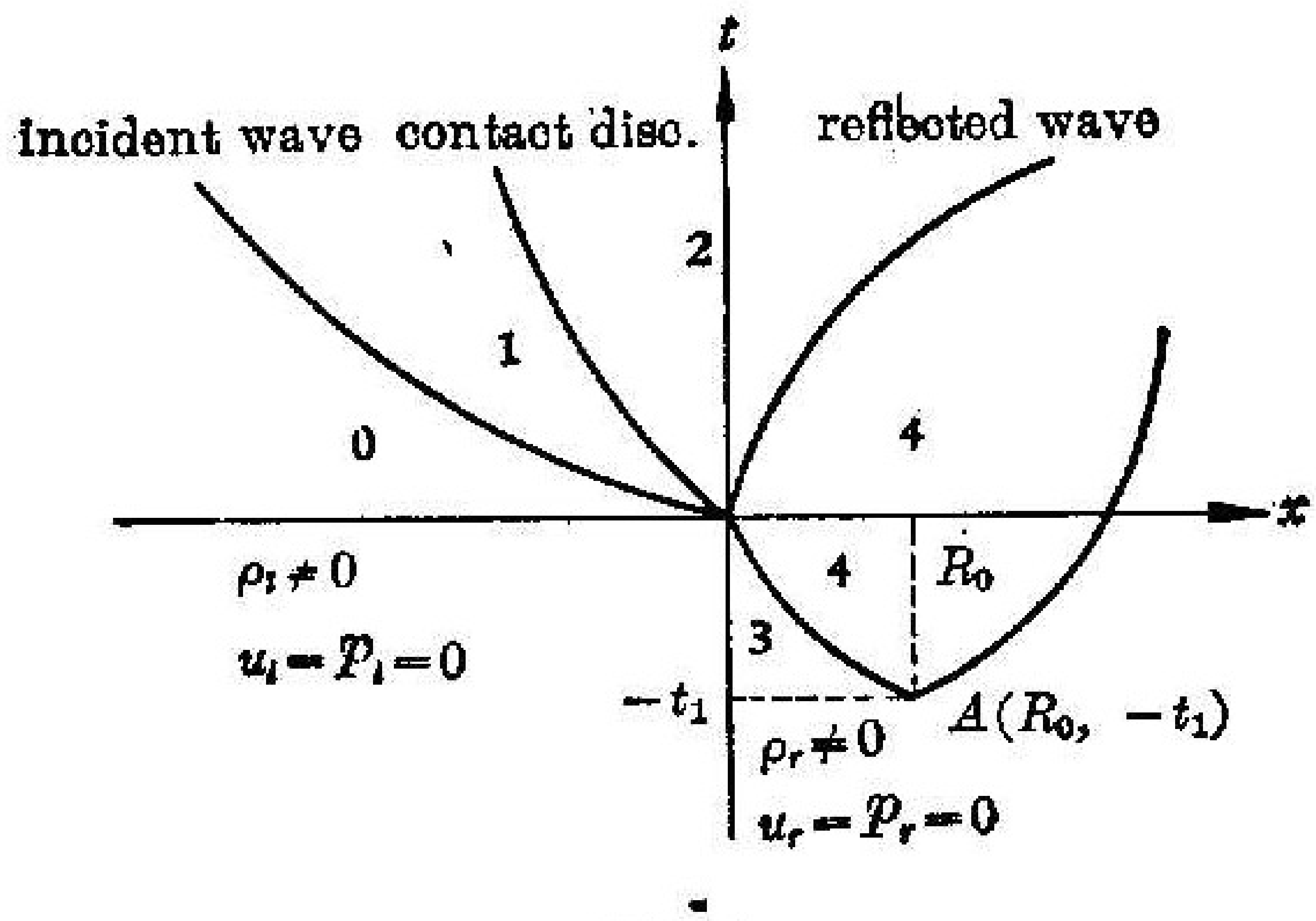

Fig. 1

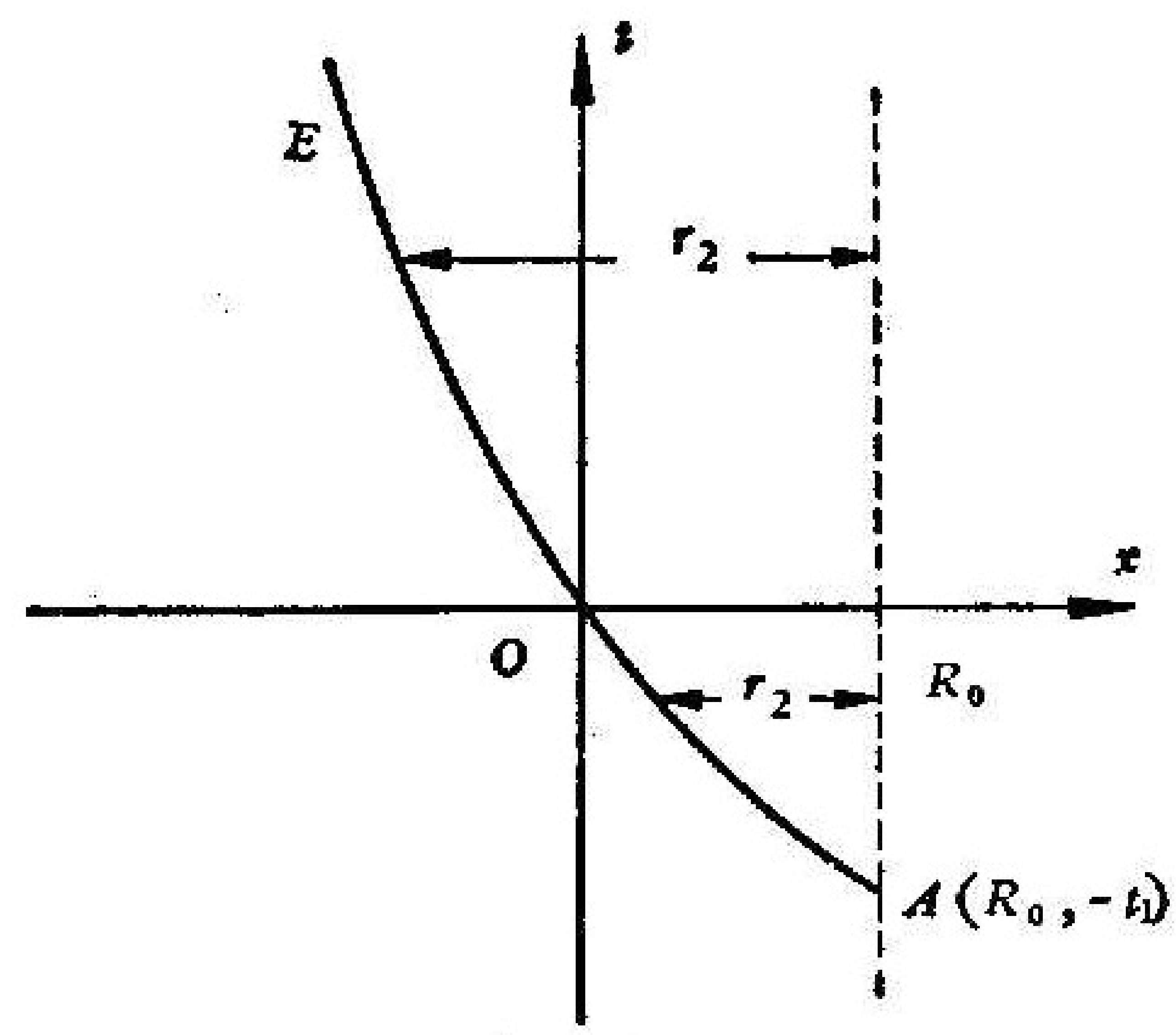

Fig. 2

at $t=0$. As a result of interaction, an incident shock, a contact discontinuity and a reflected wave (rarefaction wave or shock wave) are produced.

There is a self-similar solution in region 4 because the internal energy and the pressure of the static gas in front of the explosion wave are zero. Its analytic expressions are given in [2] and [3]. The curve AOE in Fig. 2 represents the incident shock. Let r_2 denote the distance between OE and $x=R_0$. According to those expressions, we have

$$r_2=\left(\frac{E}{\rho_r}\right)^{1/3}(t+t_1)^{2/3},$$

$$\frac{dr_2}{dt}=V=\frac{2}{3}\frac{r_2}{t+t_1}$$

(V is the velocity of explosion wave) and

$$R_0=\left(\frac{E}{\rho_r}\right)^{1/3}t_1^{2/3}.$$

Put $R_0=0.1\,\text{m}$, $E=2734905.6\,\text{J/m}^2$, $t_1=2.1718193\times10^{-5}\,\text{s}$, $\rho_0=1.29\,\text{kg/m}^3$ and the ratio of specific heats $\gamma=1.4$.

The shock relations are

$$\begin{cases}\rho_1(u_1-V)=\rho_0(u_0-V), & (2a)\\ \rho_1(u_1-V)^2+p_1=\rho_0(u_0-V)^2+p_0, & (2b)\\ e_1+\dfrac{1}{2}(u_1-V)^2+\dfrac{p_1}{\rho_1}=e_0+\dfrac{1}{2}(u_0-V)^2+\dfrac{p_0}{\rho_0}. & (2c)\end{cases}$$

Here the quantities with the subscript 0 denote the quantities in front of the wave, the quantities with the subscript 1 denote the quantities behind the wave, the internal energy $e_1=\dfrac{1}{\gamma-1}\dfrac{p_1}{\rho_1}$ and $u_0=e_0=p_0=0$. From (2) we obtain

$$\begin{cases}\rho_1(u_1-V)=-\rho_0V, & (3a)\\ \rho_1(u_1-V)^2+p_1=\rho_0V^2, & (3b)\\ \dfrac{\gamma}{\gamma-1}\dfrac{p_1}{\rho_1}+\dfrac{1}{2}(u_1-V)^2=\dfrac{1}{2}V^2. & (3c)\end{cases}$$

Furthermore, it follows from (3a—c) that

$$p_1=\rho_1u_1(V-u_1), \tag{4}$$