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A NUMERICAL EMBEDDING METHOD FOR SOLVING THE NONLINEAR COMPLEMENTARITY PROBLEM(I)——THEORY[*1)]

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Abstract

In this paper, we extend the numerical embedding method for solving the smooth equations to the nonlinear complementarity problem. By using the nonsmooth theory, we prove the existence and the continuation of the following path for the corresponding homotopy equations. Therefore the basic theory of the numerical embedding method for solving the nonlinear complementarity problem is established. In part II of this paper, we will further study the implementation of the method and give some numerical exapmles.

Key words: B-differentiable equations, Nnlinear complementarity problem, Nmerical embedding method.

1. Introduction

The nonlinear complementarity problem is a very important mathematical programming problem. The development of theory and algorithms for this problem has a long history and there have numerous methods for solving this problem. See [1] for a comprehensive review of the literature.

Recently, based on the B-differentiable equations approach, many new methods for solving the nonlinear complementarity problem have been proposed. Harker and Xiao[2] established a damped-Newton method for solving the nonlinear complementarity problem and provided many numerical results. Pang and Garbriel[7] proposed an NE/SQP method for solving the nonlinear complementarity problem and proved its global and local quadratically convergence. These methods are important to the theories and algorithms for solving the nonlinear complementarity problem.

In this paper, based on the B-differentiable equations theory, we will study: how to extend the practical numerical embedding method to the nonlinear complementarity problem; How to prove the existence, the uniqueness and the continuation of the following path for the corresponding homotopy equations by using the B-differentiable theory; How to solve the nonlinear complementarity problem by numerical embedding method proposed in this paper. All this questions will be studied in this paper and the subsequent paper.

2. Preliminaries

We consider the following nonlinear complementarity problem:
Find $x \in R^n$ such that

$$x \geq 0, \quad f(x) \geq 0, \quad and \quad x^T f(x) = 0 \tag{2.1}$$

where $f : R^n \to R^n$ satisfies $|| \bigtriangledown f(x) - \bigtriangledown f(y)|| \leq L||x-y||_0$. This problem is denoted by NCP(f).

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Following the concept of a Minty-map[2][3], the NCP(f) can be converted into the B-differentiable equations:

$$F(x) = 0 \tag{2.2}$$

where $F : R^n \to R^n$ is defined by

$$F(x) = f(x^+) + x^- \tag{2.3}$$

with $x_i^+ = \max(x_i, 0)$, $x_i^- = \min(x_i, 0)$, $x^+ = (x_1^+, x_2^+, \cdots, x_n^+)^T$ and $x^- = (x_1^-, x_2^-, \cdots, x_n^-)^T$. In other words, x solves (2.2) if and only if x^+ solves the nonlinear complementarity problem (2.1). Hence, by solving the systems (2.2), we can get the solution of the NCP(f) (2.1).

In order to present some properties of the mapping F defined by (2.3), let us review some notions in nonsmooth analysis.

The following definition is due to Robinson[13].

Definition 2.1.[6][13] *A function $F : D \subset R^n \to R^n$ is B-differentiable at a point $x \in D$ if there exists a positively homogeneous function $BF(x) : R^n \to R^n$ (i.e., $BF(x)(\lambda v) = \lambda BF(x)v$ for all $\lambda \geq 0$ and $v \in R^n$), called the B-derivative of F at x, such that*

$$\lim_{v \to 0} [F(x+v) - F(x) - BF(x)v]/\|v\| = 0.$$

If F is B-differentiable at all points $x \in D$, then F is called B-differentiable on D.

In a finite-dimensional Euclidean space R^n, Shapiro[14] showed that F is B-differentiable at x if and only if it is directionally differentiable at x. In this case, the B-derivative and the directional derivative are identical.

The basic properties of a B-differentiable function are summarized in the following theorem.

Theorem 2.2[6]. *Let $F : R^n \to R^n$ be locally Lipschitz continuous at a point x.*

(1) *If F is Fréchet differentiable at x, then it is B-differentiable at x and $BF(x) = \nabla F(x)$. Conversely, if F is B-differentiable at x and if the B-derivative $BF(x)v$ is linear in v, then F is Fréchet differentiable at x.*

(2) *If F is B-differentiable at x, then the B-derivative is unique. Moreover, $BF(x)$ is Lipschitz continuous with the same modulus as F.*

(3) *If F is B-differentiable at x, then F is directionally differentiable at x in any direction and $F'(x, d) = BF(x)d$.*

(4) *The addition, subtraction and chain rules hold for the B-derivative.*

Extending the notion of a strong F-derivative, Robinson[13] further introduced the following definition.

Definition 2.3.[13] *A function $F : D \subset R^n \to R^n$ is strong B-differentiable at a point $x \in D$ if F is B-differentiable and*

$$\lim_{\substack{y \to x \\ z \to x}} \{F(y) - F(z) - [BF(x)(y-x) - BF(x)(z-x)]\}/\|y - z\| = 0.$$

If F is strong B-differentiable at all points $x \in R^n$, then F is called strong B-differentiable on D.

Using the above definitions, it is easy to prove that[2], the function F is B-differentiable everywhere, and its B-derivative is

$$(BF(x)v)_i = \sum_{j=1}^{n} BF_i^j(x) v_j, \tag{2.4}$$

where

$$BF_i^j(x)v_j = \begin{cases} f_{ij}(x^+)v_j & j \in \alpha(x) \\ f_{ij}(x^+)v_j^+ + I_{ij}v_j^- & j \in \beta(x) \\ I_{ij}v_j & j \in \gamma(x) \end{cases}$$