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LONG TIME ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF EXPLICIT DIFFERENCE SCHEME FOR SEMILINEAR PARABOLIC EQUATIONS*1)

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Abstract

In this paper we prove that the solution of explicit difference scheme for a class of semilinear parabolic equations converges to the solution of difference schemes for the corresponding nonlinear elliptic equations in H^1 norm as $t \to \infty$. We get the long time asymptotic behavior of the discrete solutions which is interested in comparing to the case of continuous solutions.

Key words: Asymptotic behavior, Explicit difference scheme, Semilinear prarbolic equations.

1. Introduction

Let Ω be a bounded domain in R^2, $\Omega = \{0 \le x \le l, 0 \le y \le l\}$, and assume $f(x,y) \in L^\infty(\Omega), u_0(x,y) \in H^2(\Omega) \cap H_0^1(\Omega), \phi(u) \in C^1(R^1)$ satisfies

$$0 \le \phi'(u) \le \mu_1 \mid u \mid^k + \mu_2,$$

where k, μ_1 and μ_2 are positive constants.

We consider the following initial-boundary value problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \triangle u - \phi(u) + f(x,y) \quad in \quad \Omega \times R_+ \\ u \mid_{\partial\Omega} = 0 \\ u \mid_{t=0} = u_0(x,y), \quad (x,y) \in \Omega. \end{cases} \tag{1.1}$$

where $\triangle u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$ is Laplace's Operator.

By the usual approach([1],[2],[3],[4])we can get the global existence of the solution of (1.1). Furthermore, the solution of (1.1) converges to the solution of the following nonlinear elliptic equations (1.2) as $t \to \infty$.

$$\begin{cases} \triangle u - \phi(u) + f(x,y) = 0 \qquad in \quad \Omega \\ u \mid_{\partial\Omega} = 0. \end{cases} \tag{1.2}$$

In [6],[7], the authors discussed the explicit scheme for (1.1) as $f(x,y) = 0$ and only the estimate in L^2 for discrete solution was obtained. In this paper we consider the asymptotic behavior of

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discrete solution of explicit difference scheme for (1.1) which is very interested in comparing to the case of continuous solution. We also prove that the solution of explicit difference scheme for (1.1) converges to the solution of difference scheme for (1.2) as $t \to \infty$, the estimate in H^1 for discrete solution is gotten.

2. Finite Difference Scheme

Let $h, \Delta t_n$ be the space stepsize and the time stepsize respectively, $h = \frac{l}{J}$, where J is an integer. We denote the discrete function which take value w_{ij}^n at $(x_i, y_j, n\triangle t)$ $(x_i = ih, y_j = jh, 0 \le i, j \le J, n = 0, 1, 2, \cdots)$ by w_h^n, define

$$\triangle_h^1 w_{ij}^n = \frac{w_{i+1,j}^n + w_{i-1,j}^n - 2w_{ij}^n}{h^2},$$

$$\triangle_h^2 w_{ij}^n = \frac{w_{i,j+1}^n + w_{i,j-1}^n - 2w_{ij}^n}{h^2}.$$

We introduce the following notations:

$$\|w_h^n\|^2 = \sum_{0 \le i,j \le J} (w_{ij})^2 h^2,$$

$$\|\delta w_h^n\|^2 = \sum_{0 \le i,j \le J-1} [(w_{i+1,j}^n - w_{ij}^n)^2 + (w_{i,j+1}^n - w_{ij}^n)^2],$$

$$\|\triangle w_h^n\|^2 = \sum_{1 \le i,j \le J-1} (\triangle_h^1 w_{ij}^n + \triangle_h^2 w_{ij}^n)^2 h^2.$$

The explicit difference equation associate with (1.1) is:

$$\frac{u_{ij}^{n+1} - u_{ij}^n}{\Delta t_n} = \triangle_h^1 u_{ij}^n + \triangle_h^2 u_{ij}^n - \phi(u_{ij}^n) + f_{ij} \tag{2.1.1}$$

for $i, j = 1, \cdots, J-1$ and $n = 1, 2, \cdots$, where $f_{ij} = f(x_i, y_j)$. The boundary condition of (2.1) is of the form

$$u_{i,0}^n = u_{i,J}^n = u_{0,j}^n = u_{J,j}^n = 0, \quad 0 \le i, j \le J. \tag{2.1.2}$$

and the initial condition is

$$u_{ij}^0 = u_0(x_i, y_j), \quad 0 \le i, j \le J. \tag{2.1.3}$$

The difference equation corresponding to (1.2) is:

$$\triangle_h^1 u_{ij}^* + \triangle_h^2 u_{ij}^* - \phi(u_{ij}^*) + f_{ij} = 0 \tag{2.2.1}$$

for $i, j = 1, \cdots, J-1$. The boundary condition of (2.2) is of the form

$$u_{i,0}^* = u_{i,J}^* = u_{0,j}^* = u_{J,j}^* = 0, 0 \le i, j \le J. \tag{2.2.2}$$

Let the discrete function u_h^n and u_h^* be the solution of difference equation (2.1) and (2.2) respectively. For u_h^* we have the same notations to the previous. For $n = 0, 1, 2, \cdots$, the discrete function v_h^n is defined as $v_{ij}^n = u_{ij}^n - u_{ij}^*, i, j = 0, 1, \cdots, J$. Then v_h^n satisfy

$$\frac{v_{ij}^{n+1} - v_{ij}^n}{\Delta t_n} = \triangle_h^1 v_{ij}^n + \triangle_h^2 v_{ij}^n - [\phi(u_{ij}^n) - \phi(u_{ij}^*)] \tag{2.3}$$