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ABSOLUTE STABLE HOMOTOPY FINITE ELEMENT METHODS FOR CIRCULAR ARCH PROBLEM AND ASYMPTOTIC EXACTNESS POSTERIORI ERROR ESTIMATE*

Min-fu Feng Ping-bing Ming Rong-kui Yang
(*Department of Mathematics, Sichuan University 610064 P.R. China*)

Abstract

In this paper, HFEM is proposed to investigate the circular arch problem. Optimal error estimates are derived, some superconvergence results are established, and an asymptotic exactness posteriori error estimator is presented. In contrast with the classical displacement variational method, the optimal convergence rate for displacement is uniform to the small parameter. In contrast with classical mixed finite element methods, our results are free of the strict restriction on h(the mesh size) which is preserved by all the previous papers. Furtheremore we introduce an asymptotic exactness posteriori error estimator based on a global superconvergence result which is discovered in this kind of problem for the first time.

Key words: HFEM, arch, superconvergence, asymptotic exactness, posteriori error estimator

1. Introduction

Homotopy Finite Element Method(HFEM) is a new finite element method, but it's idea can be traced back to 1983 with M.Fortin. R.Glowinski's pioneering work[30], thereafter, D.N.Arnold[1] extend this method to shell problem using mixed finite element method. Tianxiao Zhou[23] applying this method to beam and Reissner-Mindlin plate model has been attained success. This method has been used by us to overcome the locking phenomenon of arch beam models recently [16], furthermore we had used the same idea to difference approximation of a nonlinear fluid bed model in some a different form [14]. Now we introduce this method in an abstract framework.

Assuming L is a differential operator, We consider the following Dirichlet problem:

$$Lu = f \quad in\ \Omega \quad u = 0 \quad on\ \Gamma$$

Ω is a bounded open domain with Lipschitz-Continuous boundary Γ for this Dirichlet problem, we consider its variational equivalent form:
Find $u \in U$ such that

$$a(u, v) = (Lu, v) = (f, v) \quad \forall v \in V$$

$$a : U \times V \to R$$

where U, V are Banach spaces, It's evident one Dirichlet problem has not one variational-equivalent form [28]. Assuming this problem has another variational-equivalent form:
Find $u \in U$ such that

$$b(u, v) = (Lu, v) = (f, v) \quad \forall v \in V$$

$$b : U \times V \to R$$

* Received.

Now, we can consider the homotopy form of a (u, v) and $b(u, v)$.

$$H(t; u, v) : [0, 1] \times U \times V \to R$$

$$H(t; u, v) = (1 - t)a(u, v) + tb(u, v)$$

Thus we have a new variational problem:
Find $u \in U$ such that

$$H(t; u, v) = (f, v) \quad \forall v \in V$$

The homotopy variational form is a triliner form with a parameter t besides the two primal variables. There are many parameter-dependent models in beam, arch, shell problems, the parameters usually are the proportion ratio of thickness and the length of beam, or the thickness of the arch, or the mid-thickness of the shell. The small parameter is the main source of locking. This kind of locking phenomenon results from lacking of K-ellipticity or having difficulty in fulfilling the Babuska-Brezzi condition. Homotopy variational principle is to construct a homotopy between two variational forms. Tianxiao Zhou and D.N.Arnold have shown the new variational form could not only enhance the K-ellipticity but also help to fulfll the Babuska-Brezzi condition. In most mechanical problem enhancing K-ellipticity is of most momentous. The method presented in this paper is a panacea to problem lacking K-ellipticity in some sense. We must remark that in most cases, the form $H(t; u, v)$ is not the real homotopy-form as defined in [27], especially in the variational principle lacks symmetry, more precisly it is only a homotopy-form in essence but not in form, this will be presented in this paper.

The circular arch model presented in this paper based upon the Timoshenko-Mindlin-Reissner assumption. The Timoshenko-Mindlin-Reissner assumption is the basis of the governing equations for this model. The key feature of this model is that the shear strain is not neglected, this assumption imposed in this kind of problem is a two folds swore, the applicable of this theory to the problem in which the thickness is not small on the one hand, but on the other hand it becomes the source of locking. For the locking phenomenon of this type, some analyse have been performed in [16,17].

A series of papers have been contributed to analyse the finite element approximation of this problem. In [2] Arnold.D.N. investigated the beam model and derived the sharp estimates, furthermore he introduced one approach to analyse this kind of problem, namely he proposed a mixed method for little parameter problem. Thereafter Kikuchi.F[9,10] carried out a detailed analysis in arch model, in [9] he analyse a arch model without shear deformation, in [10] the asymptotic expansion in terms of d(the little parameter) is presented. Recently Zhimin Zhang[25], Loula.F.D [11], Reddy.B.D.[17] presented several mixed finite element methods for these problems, optimal error estimates uniformly to the little parameter are obtained under some restrictions. In all these restrictions the restriction on the h(the mesh size) is common and that in [25] is the weakest. In [16] based on HFEM, a new variational problem is introduced to analyse the model in [25] and the uniform error estimate is obtained without any restriction on h. However the model in this paper is not the same with [25], it is the model in [11] and like the model in [17], but it is more intricate in equation and variational principle, especially it's variational principle lacks symmetry. First we introduce a new variational principle instead of the non-symmetry form in [11], Secondly as in our previous paper [16] a new mixed finite element approximation is presented by using the idea of HFEM is presented. We point out that our methods can extend to the model in [17] without any diffculty.

In this paper, HFEM is proposed to investigate the circular arch problem. Optimal error estimates are derived, some superconvergence results are establishd, and an asymptotic exactness posteriori error estimator is presented. In contrast with the classical displacement variational method, the optimal convergence rate for displacement is uniform to the small parameter. In contrast with classical mixed finite element methods, our results are free of the strict restriction on h(the mesh size) which is preserved by all the previous papers. Furtheremore we introduce