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MULTIVARIATE FOURIER SERIES OVER A CLASS OF NON TENSOR-PRODUCT PARTITION DOMAINS

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Dedicated to the 80th birthday of Professor Zhou Yulin

Abstract

This paper finds a way to extend the well-known Fourier methods, to so-called n+1 directions partition domains in n-dimension. In particular, in 2-D and 3-D cases, we study Fourier methods over 3-direction parallel hexagon partitions and 4-direction parallel parallelogram dodecahedron partitions, respectively. It has pointed that, the most concepts and results of Fourier methods on tensor-product case, such as periodicity,orthogonality of Fourier basis system, partial sum of Fourier series and its approximation behavior, can be moved on the new non tensor-product partition case.

Key words: Multivariate Fourier methods, Non tensor-product partitions, Multivariate Fourier series

1. Introduction

Fourier methods method play very important role in numerical approximation theory and its applications, e.g. see [1]. As we know, the original result has been studied in univariate case. Strictly, the tensor product approach is still staying in the one dimension level via decreasing dimension. How to generalize the approach into higher dimension, beyond box domains, is still an open problem. On an equilateral triangle case, Pinsky in 1980 [2] and 1985 [3] and Prágerin 1998 [4] have studied eigen-decompositions of the Laplace operator as generalized Fourier transformation. Recently Sun [5]-[7] has constructed a partial foundation to define generalized Fourier transformation on an arbitrary triangular domain also via eigen-decomposition. It is well-known that a triangle in 2-D and a simplex in 3-D are natural non-box extensions of the interval $[0, 1]$ in 1-D, and the origin Fourier transformation is carried on the interval $[-1, 1]$. It seems there is no essential difference between intervals $[0, 1]$ and $[-1, 1]$ in 1-D, however, the situation is quite different in high dimension. What is more natural non-box extensions in 2-D and 3-D of the interval $[-1, 1]$ in 1-D? In this paper we point that a parallel hexagon and a parallel dodecahedron can be as a direct generalization in 2-D and 3-D of the symmetry interval $[-1, 1]$, respectively. In next sections at first we introduce 3-direction and 4-direction mesh in 2-D and 3-D, respectively. Then we define a parallel hexagon in 2-D, and a parallel hexagon prism and a parallel dodecahedron in 3-D as our three basic periodic domains. Finally we proposed an orthogonal basis system on related function space. We have proposed that the most concepts and results of Fourier methods on tensor-product case, such as periodicity, orthogonality of Fourier basis system, the related sine and cosine transformations, partial sum

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of Fourier series, discreting Fourier transformation (DFT), Fast Fourier transformation (FFT) and its approximation behavior, can be moved on the new non tensor-product partition case.

2. A basic function system on 3-direction partition

Given an origin point O and two plane vectors e_1 and e_2, we form a 3-direction 2-D partition as drawn in Fig. 1. To deal with symmetry along the three direction, we adapt a 3-direction coordinates instead of the usual two coordinates. Setting the origin point $O=(0,0,0)$, each partition line is represented by t_l=integer (l=1,2,3), and each 2-D point P is represented by

$$P=(t_1,t_2,t_3), \quad t_1+t_2+t_3=0, \tag{2.1}$$

and any function $f(P)$ defined on the plane can be written as

$$f(P)=f(t_1,t_2,t_3), \quad t_1+t_2+t_3=0$$

In particulary, P_k is called an integer node if and only if for an integer pair

$$P_k=(k_1,k_2,k_3), \quad k_1+k_2+k_3=0.$$

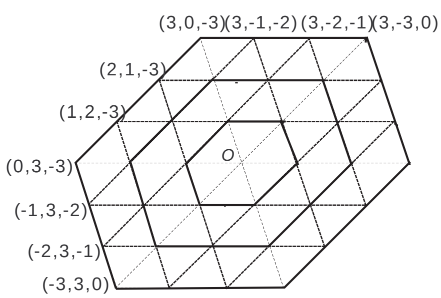

Fig.1: 3-direction partition

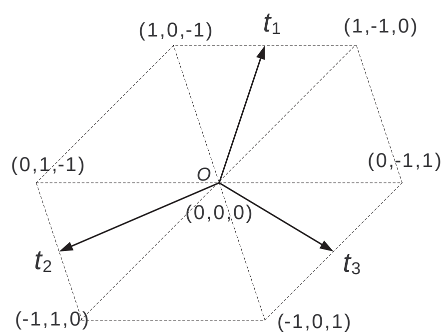

Fig.2:Parallel hexagon domain Ω

Definition 2.1. *A function $f(P)$, defined in the 3-direction coordinate, is called periodic with period $Q=(\tau_1,\tau_2,\tau_3), \tau_1+\tau_2+\tau_3=0$, if for all $P=(t_1,t_2,t_3), t_1+t_2+t_3=0$*

$$f(P+Q)=f(P)$$

We take the following parallel hexagon Ω, drawn in Fig.2, as our basic domain

$$\Omega=\{P|P=(t_1,t_2,t_3) \qquad t_1+t_2+t_3=0, \quad -1\le t_1,t_2,t_3\le 1\} \tag{2.2}$$

Lemma 2.1. *For any integer pair (n_1,n_2,n_3) with $n_1+n_2+n_3=0$, then*

$$n_1-n_2=n_2-n_3=n_3-n_1=\nu, \qquad (mod \;\; 3) \quad (\nu=-1,0,1)$$

and

$$n_1^2+n_2^2+n_3^2=2\nu^2 \qquad (mod \;\; 6), \quad (\nu=0,1)$$

Definition 2.2. *For a given integer pair j=(j_1,j_2,j_3) with $j_1+j_2+j_3=0$,let $\omega=e^{i\frac{2\pi}{3}}$, a complex function $g_j(P)$ on the three-direction coordinates is defined*

$$g_j(P)=\omega^{j_1t_1+j_2t_2+j_3t_3}. \tag{2.3}$$