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CONIC TRUST REGION METHOD FOR LINEARLY CONSTRAINED OPTIMIZATION*1)

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Abstract

In this paper we present a trust region method of conic model for linearly constrained optimization problems. We discuss trust region approaches with conic model subproblems. Some equivalent variation properties and optimality conditions are given. A trust region algorithm based on conic model is constructed. Global convergence of the method is established.

Key words: Trust region method, Conic model, Constrained optimization.

1. Introduction

Trust region methods have very nice global and local convergence properties, and it has been shown that they are very effective and robust for solving unconstrained and constrained optimization problems (for example, see [2], [3], [4], [6] , [10], [11], [13], [14], [15], [17], [19] and [27]). Conic model methods, a generalization of quadratic model methods, possess more degree of freedom, can incorporate more information in the iterations, and provide both a powerful unifying theory and an effective means for optimization problems [1] [4] [5] [12] [18] [22] [26].

In [4], a trust region method of conic model for unconstrained optimization problems was presented. It is shown that this method is advantageous in both theory and numerical aspects. In this paper, we further describe a trust region method of conic model to solve linearly constrained optimization problem

$$\min \quad f(x) \tag{1.1}$$

$$\text{s.t.} \quad A^T x = b, \tag{1.2}$$

where $f : R^n \to R$ is continuously differentiable, $A \in R^{n\times m}, x \in R^n, b \in R^m, \text{rank}(A) = m$. Our method is iterative, and the trust region subproblem solved in each iteration is the minimization of a conic model subject to the linear constraints and an additional trust region constraint.

Normally, numerical methods for solving optimization problem (1.1)-(1.2) are reduced gradient method, projected gradient method and reduced quasi-Newton method which are based on quadratic model. Using null space techniques, the constrained problem (1.1)-(1.2) can be transformed to an unconstrained problem. In order to incorporate more useful interpolation information in constructing subproblems, Davidon [5] suggested a new model – conic model. A typical conic model for unconstrained optimization is as follows:

$$\psi(s) = f_k + \frac{g_k^T s}{1 - a^T s} + \frac{1}{2}\frac{s^T A_k s}{(1 - a^T s)^2}, \tag{1.3}$$

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where $f_k = f(x_k), g_k = \nabla f(x_k), A_k \in R^{n\times n}$ is a symmetric matrix, the vector $a \in R^n$ is a vector satisfying $1 - a^T s > 0$. If $a = 0, \psi(s)$ is reduced to be quadratic. Also note that $\psi(s)$ is quadratic along any direction $s \in R^n$ satisfying $a^T s = 0$.

The conic model (1.3) can be also written as the following form of the collinear conic model:

$$\psi(s) = f_k + g_k^T w + \frac{1}{2} w^T A_k w \tag{1.4}$$

$$s = \frac{w}{1 + a^T w}. \tag{1.5}$$

It follows from (1.4)-(1.5) that

$$s = \frac{-A_k^{-1} g_k}{1 - a^T A_k^{-1} g_k}$$

is a minimizer of $\psi(s)$ if A_k is positive definite.

Sorensen [18] discussed collinear scaling methods for unconstrained optimization. For the scaling function

$$\phi_{k+1}(w) = f(\bar{x}(w)) = f(x_{k+1} + \frac{w}{1 + h_{k+1}^T w}), \tag{1.6}$$

the corresponding quadratic model is

$$\psi_{k+1}(w) = \phi_{k+1}(0) + \phi'_{k+1}(0) w + \frac{1}{2} w^T B_{k+1} w, \tag{1.7}$$

which satisfies the following interpolation conditions

$$\psi_{k+1}(0) = \phi_{k+1}(0),\ \psi'_{k+1}(0) = \phi'_{k+1}(0), \tag{1.8}$$

$$\psi_{k+1}(-v) = \phi_{k+1}(-v),\ \psi'_{k+1}(-v) = \phi'_{k+1}(-v), \tag{1.9}$$

where $v \in R^n$ is chosen such that

$$1 - h_{k+1}^T v > 0. \tag{1.10}$$

Di and Sun [4] consider a trust region method of conic model for unconstrained optimization. They give the following model

$$\min \quad \psi(s) = f_k + \frac{g_k^T s}{1 - a^T s} + \frac{1}{2} \frac{s^T B_k s}{(1 - a^T s)^2} \tag{1.11}$$

$$\text{s.t.} \quad \|Ds\| \le \Delta_k \tag{1.12}$$

or equivalently

$$\min \quad f_k + g_k^T J_k w + \frac{1}{2} w^T B_k w \tag{1.13}$$

$$\text{s.t.} \quad s = \frac{J_k w}{1 + h^T w},\ \|Ds\| \le \Delta_k. \tag{1.14}$$

They construct a trust region algorithm based on the above model, and give convergence analyses. Another class of conic trust region methods for unconstrained optimization is presented in [8] where the model is

$$\min \quad \psi(s) = f_k + \frac{g_k^T s}{1 - a^T s} + \frac{1}{2} \frac{s^T B_k s}{(1 - a^T s)^2} \tag{1.15}$$

$$\text{s.t.} \quad \left\|\frac{s}{1 - a^T s}\right\| \le \Delta_k. \tag{1.16}$$

This method with self-adjust strategy has been studied and has desired numerical results.

In this paper we generalize the trust region method of conic model for unconstrained optimization to solve linearly constrained optimization problem (1.1)-(1.2). In Section 2, the motivation and a detailed description of our method are given. Convergence analyses of the new algorithm are presented in Section 3. In addition, the conic trust region method for nonlinearly constrained optimization is also presented separately in [25].