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PARALLEL DYNAMIC ITERATION METHODS FOR SOLVING NONLINEAR TIME-PERIODIC DIFFERENTIAL-ALGEBRAIC EQUATIONS*[1)]

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Abstract

In this paper we presented a convergence condition of parallel dynamic iteration methods for a nonlinear system of differential-algebraic equations with a periodic constraint. The convergence criterion is decided by the spectral expression of a linear operator derived from system partitions. Numerical experiments given here confirm the theoretical work of the paper.

Key words: Nonlinear dynamic equations, Periodic solutions, Dynamic iterations, Engineering applications.

1. Introduction

In order to analyze physical characters of nonlinear dynamic equations issued from engineering applications we often need to compute their periodic solutions. Most models of mechanical systems and circuit simulation might be described by nonlinear differential-algebraic equations (DAEs) as follows

$$\left\{ \begin{array}{l} \dfrac{d}{dt}x(t) = \tilde{f}(x(t), y(t), t), \quad x(0) = x(T), \\ y(t) = \tilde{g}(x(t), y(t), t), \quad t \in [0, T], \end{array} \right. \tag{1}$$

where $x(t) \in \mathbf{R}^n$ and $y(t) \in \mathbf{R}^m$ for $t \in [0,T]$, $\tilde{f} : \mathbf{R}^n \times \mathbf{R}^m \times [0,T] \mapsto \mathbf{R}^n$ and $\tilde{g} : \mathbf{R}^n \times \mathbf{R}^m \times [0,T] \mapsto \mathbf{R}^m$ satisfy $\tilde{f}(x,y,0) = \tilde{f}(x,y,T)$ and $\tilde{g}(x,y,0) = \tilde{g}(x,y,T)$ for any given $x \in \mathbf{R}^n$ and $y \in \mathbf{R}^m$, and $y(0)$ satisfies $y(0) = \tilde{g}(x(0), y(0), 0)$.

It is a general knowledge that computation of periodic solutions for a dynamic system is very time-consuming owing to the unknown of initial values. The usual way to treat (1) is the shooting techniques and its variants [1, 2]. For example, if the time interval is broken into small pieces then the parallel shooting process is available. This parallel process is not direct since the standard shooting must be called for every small interval.

A direct parallel method for transient computation is dynamic iteration (see [3]) or waveform relaxation. The dynamic iteration method was originally presented to simulate VLSI in 1982 [4]. It decouples dynamic equations in system level, for example a system of ordinary differential equations (ODEs) or DAEs may be partitioned into some simplified systems of ODEs or DAEs [5, 6, 7]. We can also study dynamic iterations of linear integral-differential-algebraic equations (IDAEs) [8]. Numerical algorithms based on this approach could be conveniently implemented on multi-processor computer systems [9]. They are instinctive parallel algorithms.

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The dynamic iteration method has been adopted as a computational tool to study periodic solutions of linear systems [10, 11]. A simple form of nonlinear ODEs under a periodic excitation is studied in [12]. There is no theoretical work in this field to analyze the case of nonlinear DAEs as (1). In this paper for a class of nonlinear functions $\tilde{f}$ and $\tilde{g}$ we give a convergence condition for the dynamic iteration solutions of (1). The criterion comes from an analytically spectral expression of a linear operator. The linear operator is a periodically dynamic iteration operator resulted in system partitions. A nonlinear DAEs example is provided to illustrate the novel condition.

2. Main Results

The parallel dynamic iteration method of (1) is written as

$$\begin{cases} \frac{d}{dt}x^{(k+1)}(t) = f(x^{(k+1)}(t), x^{(k)}(t), y^{(k+1)}(t), y^{(k)}(t), t), \quad x^{(k+1)}(0) = x^{(k+1)}(T), \\ y^{(k+1)}(t) = g(x^{(k+1)}(t), x^{(k)}(t), y^{(k+1)}(t), y^{(k)}(t), t), \quad t \in [0,T], \quad k = 0, 1, \ldots, \end{cases} \tag{2}$$

where the function $x^{(0)}(t)$ is an initial guess with $x^{(0)}(0) = x^{(0)}(T)$ and the nonlinear splitting functions $f : (\mathbf{R}^n)^2 \times (\mathbf{R}^m)^2 \times [0,T] \mapsto \mathbf{R}^n$ and $g : (\mathbf{R}^n)^2 \times (\mathbf{R}^m)^2 \times [0,T] \mapsto \mathbf{R}^m$ satisfy

$$f(x, x, y, y, t) = \tilde{f}(x, y, t), \quad g(x, x, y, y, t) = \tilde{g}(x, y, t), \quad t \in [0,T], \tag{3}$$

in which $x \in \mathbf{R}^n$ and $y \in \mathbf{R}^m$. Typical and important partitions in practical application are the Jacobi and Gauss-Seidel splitting functions.

We let $\mathbf{R}^l$ $(l = m, n)$ be the standard Euclid space equipped with an inner product $\langle \cdot, \cdot \rangle$. The 2-norm $\| \cdot \|$ in $\mathbf{R}^l$ is induced by the inner product. For the splitting functions f and g in (3) we assume that they obey the following condition.

Condition (L). *(1) For $f(\cdot, u_2, u_3, u_4, t)$, on $[0,T]$ there is a positive constant a_1 such that*

$$\langle f(u_1, u_2, u_3, u_4, t) - f(v_1, u_2, u_3, u_4, t), u_1 - v_1 \rangle \leq -a_1 \|u_1 - v_1\|^2, \; u_1, v_1 \in \mathbf{R}^n; \tag{4}$$

(2) For $f(u, \cdot, \cdot, \cdot, t)$, on $[0,T]$ there are nonnegative constants a_j $(j = 2, 3, 4)$ such that

$$\|f(u, u_2, u_3, u_4, t) - f(u, v_2, v_3, v_4, t)\| \leq \sum_{j=2}^{4} a_j \|u_j - v_j\|, \; u_2, v_2 \in \mathbf{R}^n, \; u_l, v_l \in \mathbf{R}^m (l = 3, 4); \tag{5}$$

(2) For $g(\cdot, \cdot, \cdot, \cdot, t)$, on $[0,T]$ there are nonnegative constants b_j $(j = 1, 2, 3, 4)$ such that

$$\begin{aligned} &\|g(u_1, u_2, u_3, u_4, t) - g(v_1, v_2, v_3, v_4, t)\| \\ &\leq \sum_{j=1}^{4} b_j \|u_j - v_j\|, \; u_l, v_l \in \mathbf{R}^n (l = 1, 2), \; u_s, v_s \in \mathbf{R}^m (s = 3, 4). \end{aligned} \tag{6}$$

The inequality of (4) is a strongly dissipative condition. The inequalities (5) and (6) are classical Lipschitz conditions. In this paper we assume that (1) and each approximative system in (2) have periodic solutions.

We denote that $\widetilde{b}_1 = \frac{b_1}{1 - b_3}$, $\widetilde{b}_2 = \frac{b_2}{1 - b_3}$, and $\widetilde{b}_4 = \frac{b_4}{1 - b_3}$. Further, we also let $\widetilde{a}_1 = a_1 - a_3\widetilde{b}_1$, $\widetilde{a}_2 = a_2 + a_3\widetilde{b}_2$, and $\widetilde{a}_4 = a_4 + a_3\widetilde{b}_4$. For $w \in C([0,T], \mathbf{C})$ or $L^2([0,T], \mathbf{C})$, we define a linear operator $\mathcal{R}$ as

$$(\mathcal{R}w)(t) = e^{-\widetilde{a}_1 t}(1 - e^{-\widetilde{a}_1 T})^{-1} \int_0^T e^{-\widetilde{a}_1 (T-s)} w(s) ds + \int_0^t e^{-\widetilde{a}_1 (t-s)} w(s) ds, \quad t \in [0,T]. \tag{7}$$