Journal of Computational Mathematics, Vol.21, No.3, 2002, 375–382.

STABILITIES OF (A,B,C) AND NPDIRK METHODS FOR SYSTEMS OF NEUTRAL DELAY-DIFFERENTIAL EQUATIONS WITH MULTIPLE DELAYS *

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Abstract

Consider the following neutral delay-differential equations with multiple delays (NMDDE)

$$y'(t) = Ly(t) + \sum_{j=1}^{m}[M_j y(t-\tau_j) + N_j y'(t-\tau_j)], \quad t \geq 0, \tag{0.1}$$

where $\tau > 0, L, M_j$ and N_j are constant complex-value $d \times d$ matrices. A sufficient condition for the asymptotic stability of NMDDE system (0.1) is given. The stability of Butcher's (A,B,C)-method for systems of NMDDE are studied. In addition, we present a parallel diagonally-implicit iteration RK (PDIRK) methods(NPDIRK) for systems of NMDDE, which is easier to be implemented than fully implicit RK methods. We also investigate the stability of a special class of NPDIRK methods by analyzing their stability behaviors of the solutions of (0.1).

Key words: Neutral delay differential equations, (A,B,C)-method, RK method, Parallel diagonally-implicit iteration RK method.

1. Introduction

Consider the stability behavior in the numerical solution of neutral delay differential equations with multiple delays(NMDDE)

$$y'(t) = f(t, y(t), y(t-\tau_1), \cdots, y(t-\tau_m), y'(t-\tau_1), \cdots, y'(t-\tau_m)), \ t \geq 0, \tag{1.1}$$

$$y(t) = g(t), \qquad -\tau \leq t \leq 0, \tag{1.2}$$

where τ_j are some given positive constants for $j = 1, \cdots, m$, $\tau_m > \tau_{m-1} > \cdots > \tau_1 > 0$, f and g denote given vector-value functions and $y(t)$ is the unknown function to be solved for $t \geq 0$. The purpose of the present work is to investigate the stability properties of Butcher's (A,B,C) (see [2]) and NPDIRK methods by means of the linear test equations of the type (1.1), i.e.

$$\begin{cases} y'(t) = Ly(t) + \sum_{j=1}^{m}[M_j y(t-\tau_j) + N_j y'(t-\tau_j)], \quad t \geq 0, \\ y(t) = g(t), \ -\tau \leq t \leq 0, \end{cases} \tag{1.3}$$

where $\tau > 0, L, M_j$ and N_j are constant complex-value $d \times d$ matrices. The methods represented by matrices (A,B,C) are called as general linear method by Butcher(see([2]). It includes many numerical methods such as RK and LM methods and so on. As an example of (A,B,C) methods, we also obtain the stability result of RK methods. NPDIRK method is a new scheme for numerical solving NMDDE that is presented in this paper.

There have been a number of studies on the numerical stability analysis of system (1.3) with the cases of $m = 1$ and/or $d = 1$ and/or $N_j = 0$. The stability of RK and one-parameter

* Received January 15, 2001; final revised March 13, 2002.

methods for the case $m=1$ have been investigated in [10,11]. [1] and [9] have studied stability properties of RK methods for $m=1$ and $d=1$. The stability analysis of RK methods and linear multistep methods for system (1.3) with $N_j=0$ $(j=1,\cdots,m)$ and/or $\tau_j=j\tau$, has been studied in [5,6]. In [15] Zhang & Zhou studied the numerical stability of multistep RK methods for system (1.3).

In this paper, we firstly give a sufficient condition for the asymptotic stability of the test NMDDE (1.3). Then, we extend and generalize the stability results of [1,5,6,9,10,11,15] to the general (A,B,C) methods for numerical solving NMDDE (1.1)(1.2). Finally, in section 5, we present NPDIRK scheme for NMDDE (1.1)(1.2), which is easier to be implemented than fully implicit RK methods on parallel computers for numerically solving NMDDE (1.1) and (1.2). We also study stability properties of a special class of PDIRK methods with respect to (1.3).

2. The Asymptotic Stability of Test Equation (1.3)

Definition 2.1. *The NMDDE (1.3) is called to be asymptotically stable, if for any continuous differentiable initial function $g(t)$ and for any delay $\tau_j>0$, $(\tau_m>\tau_{m-1}>\cdots>\tau_1)$, the solution of (1.3) $y(t)\to 0$ as $t\to\infty$.*

Let

$$Q(v_1,v_2,\cdots,v_m):=(I-\sum_{j=1}^{m}N_jv_j)^{-1}(L+\sum_{j=1}^{m}M_jv_j), \tag{2.1}$$

$\lambda_i(F)$ and $Re\lambda_i(F)$ stand for the i-th eigenvalue and the real part of i-th eigenvalue, respectively, of any matrix F.

Lemma 2.1. *The system (1.3) is asymptotically stable if the following conditions*

$$\sup Re\lambda_i(Q(v_1,v_2,\cdots,v_m))<0, \quad for \ \ all \ \ i \ \ and \ \ v_j\in C \ and \ |v_j|\le 1 \tag{2.2}$$

and

$$\sum\nolimits_{j=1}^{m}\| N_j \|<1 \tag{2.3}$$

hold, where $||N_j||=\sup\limits_{||\xi||=1}||N_j\xi||$.

The proof is analogous to that of Theorem in [15] we omit it here.

3. Stability of (A,B,C)-Methods for System (1.3)

For the initial-value problem of ODEs

$$y'(t)=f(t,y(t)), \quad t\ge 0, \quad y(0)=y_0, \tag{3.1}$$

An (A,B,C)-method for (3.1) is given in a standard form as (see [2])

$$Y_{n+1}=(A\otimes I)Y_n+h(B_0\otimes I)F_n+h(B_1\otimes I)F_{n+1}, \tag{3.2}$$

where I stands for the identity matrix; $A,B_0,B_1\in R^{r\times r}, Y_{n+1}=(y_{n,1}^T,y_{n,2}^T,\cdots,y_{n,r}^T)^T$, $y_{n,i}\approx y(t_n+\alpha_ih)$, $\alpha_i>0$, $\alpha_i\ne\alpha_j$ (if $i\ne j$), $F_{n+1}=(f^T(t_n,y_{n,1}),\cdots,f^T(t_n,y_{n,r}))^T$. Define

$$r(\overline{h}):=(I-\overline{h}B_1)^{-1}(A+\overline{h}B_0) \tag{3.3}$$

as the stability matrix of the (A,B,C) method. It is well known that (A,B,C)-method (3.2) is said to be A-stable for ODE (3.1) if

$$(I-\overline{h}B_1) \text{ is regular and } \rho(r(\overline{h}))<1 \text{ for any } Re(\overline{h})<0. \tag{3.4}$$