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THE GLOBAL ARTIFICIAL BOUNDARY CONDITIONS FOR NUMERICAL SIMULATIONS OF THE 3D FLOW AROUND A SUBMERGED BODY *1)

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Abstract

We consider the numerical approximations of the three-dimensional steady potential flow around a body moving in a liquid of finite constant depth at constant speed and distance below a free surface in a channel. One vertical side is introduced as the upstream artificial boundary and two vertical sides are introduced as the downstream artificial boundaries. On the artificial boundaries, a sequence of high-order global artificial boundary conditions are given. Then the original problem is reduced to a problem defined on a finite computational domain, which is equivalent to a variational problem. After solving the variational problem by the finite element method, we obtain the numerical approximation of the original problem. The numerical examples show that the artificial boundary conditions given in this paper are very effective.

Key words: Ship wave, Potential flow, Global artificial boundary condition, Finite element method.

1. Introduction

Consider the three-dimensional steady potential flow around a body moving in a liquid of finite constant depth at constant speed and distance below a free surface in a channel. Let d denote the depth of the liquid, c denote the width of the channel, U denote the speed of the body and g denote the acceleration of gravity. We scale the physical quantities by the length d and the velocity $\sqrt{gd}$. We describe the motion in Cartesian coordinates fixed with respect to the body, where the x-axis points opposite to the forward velocity and z-axis is directed vertically upward, y-axis points the remaining direction of the right-angle reference frame, y=0 corresponds to one side of the channel and y=c to another side of the channel, z=0 corresponds to the undisturbed free surface and z=-1 to the bottom. Let Ω_i denote the domain occupied by the body, then $\Omega = \{\mathbb{R}\times(0,c)\times(-1,0)\}\backslash\bar{\Omega}_i$ is the domain occupied by the liquid. The total velocity potential is split into a free stream potential plus a perturbation potential: $\Phi{=}\mu x+\phi(x,z)$, where $\mu{=}U/\sqrt{gd}$ is the Froude number. By linearizing the boundary condition at the free surface, see whitham[22], we obtain the following problem for the perturbation potential on the unbounded domain Ω:

$$\triangle\phi = 0 \qquad \text{in } \Omega, \tag{1.1}$$

together with the boundary conditions

$$(\mu^2\phi_{xx}+\phi_z)|_{z=0} = 0 \qquad -\infty < x < +\infty, 0 < y < c \tag{1.2}$$

$$\phi_z|_{z=-1} = 0 \qquad -\infty < x < +\infty, 0 < y < c \tag{1.3}$$

$$\phi_y|_{y=0} = 0 \qquad -\infty < x < +\infty, -1 < z < 0 \tag{1.4}$$

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$$\phi_y|_{y=c} = 0 \qquad -\infty < x < +\infty, -1 < z < 0 \tag{1.5}$$

$$\frac{\partial \phi}{\partial n} = \mu cos\theta \qquad \partial\Omega_i, \tag{1.6}$$

$$\lim_{x\to-\infty} \phi = 0, \lim_{x\to+\infty} \phi \text{ is bounded} \qquad -1 < z < 0; \tag{1.7}$$

where $\partial/\partial n$ denote the outward normal derivative of Ω, in the following $\partial/\partial n$ always denote the outward normal derivative of a given domain. θ is the angle between the outwardly directed normal to the body and the x-direction.

There are many authors who studied the numerical simulations of the flow around a submerged body in two dimensional case. For examples, Petersson and Malmliden [20] studied the numerical solutions of the given 2-D problem using composite grids, furthermore Malmliden and Petersson [17] proposed a Schwarz-type iterative method. Doctors and Beck [3], Nakos and Sclavounos [18] presented the boundary integral methods. In this paper we will concentrate on the numerical simulations of the 3D flow around a submerged body by the artificial boundary method. The artificial boundary method is very popular used for overcoming the difficulty caused by the unboundedness of the physical domain. During the last two decades, there are many mathematicians and engineers who have worked on this field for various problems by different techniques, see references [4]-[15], [21], [23].

For the given problem (1.1)-(1.7), we introduce the upstream artificial boundary Γ_a, the downstream artificial boundary Γ_b and the auxiliary artificial boundary $\Gamma_{b'}$. We design the high-order artificial boundary conditions on Γ_a, Γ_b and $\Gamma_{b'}$, then the given problem (1.1)-(1.7) is reduced to a boundary value problem on bounded computational domain, which can be solved by the finite element method. Furthermore the numerical example shows the effectiveness of the method given in this paper.

2. The Global Artificial Boundary Conditions

Take three constants $a < b' < b$, such that $\Omega_i \subset (a, b') \times (0, c) \times (-1, 0)$. Then we obtain the upstream artificial boundary $\Gamma_a = \{(x, y, z) : x = a, 0 \le y \le c, -1 \le z \le 0\}$, the downstream artificial boundary $\Gamma_b = \{(x, y, z) : x = b, 0 \le y \le c, -1 \le z \le 0\}$, and the auxiliary artificial boundary $\Gamma_{b'} = \{(x, y, z) : x = b', 0 \le y \le c, -1 \le z \le 0\}$. The artificial boundaries Γ_a, Γ_b divide the domain Ω into three parts:

$$\Omega_a = \{(x, y, z) : -\infty < x < a, 0 < y < c, -1 < z < 0\},$$
$$\Omega_T = \{(x, y, z) : a < x < b, 0 < y < c, -1 < z < 0\}\backslash\overline{\Omega_i},$$
$$\Omega_b = \{(x, y, z) : b < x < +\infty, 0 < y < c, -1 < z < 0\},$$

furthermore we denote

$$\Omega_{b'} = \{(x, y, z) : b' < x < +\infty, 0 < y < c, -1 < z < 0\}.$$

2.1. The Artificial Boundary Condition on the Downstream Artificial Boundary

We consider the artificial boundary condition on the downstream artificial boundary. The restriction of the solution of the problem (1.1)-(1.7) on the domain $\Omega_{b'}$ satisfies:

$$\triangle\phi = 0 \qquad \text{in } \Omega_{b'}, \tag{2.1}$$

$$(\mu^2\phi_{xx} + \phi_z)|_{z=0} = 0 \qquad b' < x < +\infty, 0 < y < c \tag{2.2}$$

$$\phi_z|_{z=-1} = 0 \qquad b' < x < +\infty, 0 < y < c \tag{2.3}$$

$$\phi_y|_{y=0} = 0 \qquad -\infty < x < +\infty, -1 < z < 0 \tag{2.4}$$

$$\phi_y|_{y=c} = 0 \qquad -\infty < x < +\infty, -1 < z < 0 \tag{2.5}$$

$$\lim_{x\to+\infty} \phi \text{ is bounded}; \tag{2.6}$$

where the domain $\Omega_{b'}$ is a semi-infinite channel. The problem (2.1)-(2.6) is an uncompletely posed problem. The general solution of problem (2.1)-(2.6) is given in [16] by the separation of