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GLOBAL SUPERCONVERGENCE OF THE MIXED FINITE ELEMENT METHODS FOR 2-D MAXWELL EQUATIONS *1)

Jia-fu Lin
(*Institute of Systems Sciences, Chinese Academy of Sciences, Beijing 100080, China*)

Qun Lin
(*LSEC, ICMSEC, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing 100080, China*)

Abstract

Superconvergence of the mixed finite element methods for 2-d Maxwell equations is studied in this paper. Two order of superconvergent factor can be obtained for the k-th Nedelec elements on the rectangular meshes.

Key words: Maxwell equations, Mixed finite element, Superconvergence, Postprocessing.

1. Introduction

Superconvergence of the mixed finite element methods for 3-d Maxwell equations was first considered by Monk [8]. In 1999, Lin and Yan [4] used the integral identity technique to study this problem once more and improved Monk's result. One order of superconvergent factor was obtained by them for k-th Nedelec elements on the cubic meshes. The similar result was proved for 2-d Maxwell equations by Lin and Yan [5] and Brandts [1]. In this paper, we improve the Brandts' result. If the domain is rectangular, two order of superconvergent factor which is one order higher than Brandts' result can be obtained for the k-th ($k \geq 1$) Nedelec elements on the rectangular meshes.

The paper is organized as follows: In section 2, the mixed finite element formulation for solving 2-d Maxwell equations is introduced. In section 3, we will consider the k-th ($k \geq 1$) Nedelec elements on the rectangular meshes and prove some basic estimates. In section 4, the mixed elliptic projection operator is defined and the error between the interpolation operator and the projection operator is estimated by utilizing the method introduced in [1]. In section 5, we obtain the superclose result. In the last section, the global superconvergence is obtained by the postprocessing.

2. Formulation

Consider the following two-dimension Maxwell equations

$$\mathbf{E}_t - \mathbf{rot} H = -\mathbf{J} \qquad \text{in } \Omega \times (0,T), \tag{1}$$

$$H_t + \mathrm{curl}\mathbf{E} = 0 \qquad \text{in } \Omega \times (0,T), \tag{2}$$

$$\mathbf{n} \times \mathbf{E} = 0 \qquad \text{on } \partial\Omega \times (0,T), \tag{3}$$

$$\mathbf{E}(0) = \mathbf{E}_0, \quad H(0) = H_0, \tag{4}$$

where $\mathbf{E} = (E_1, E_2)$, $\mathbf{rot} H = (\frac{\partial H}{\partial y}, -\frac{\partial H}{\partial x})$, $\mathrm{curl}\mathbf{E} = \frac{\partial E_2}{\partial x} - \frac{\partial E_1}{\partial y}$, $\mathbf{n} \times \mathbf{E} = E_2 n_1 - E_1 n_2$, $\mathbf{n} = (n_1, n_2)$ is the unit outward norm of $\partial\Omega$, $\Omega \subset \mathbf{R}^2$ is a bounded domain. In the following, we will use

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the notations

$$\|\cdot\|_0,\ \|\cdot\|_{0,e}\ \text{ for }\ L^2(\Omega),\ L^2(e)-\text{norm},$$

and

$$\|\cdot\|_k,\ \|\cdot\|_{k,e}\ \text{ for }\ H^k(\Omega),\ H^k(e)-\text{norm}.$$

Let

$$\mathbf{H}_0(\text{curl};\Omega)=\{\mathbf{v}=(v_1,v_2)\in(L^2(\Omega))^2;\text{curl}\mathbf{v}\in L^2(\Omega),\mathbf{n}\times\mathbf{v}\mid_{\partial\Omega}=0\},$$

with norm

$$\|\mathbf{v}\|_{\mathbf{H}(\text{curl};\Omega)}=\{\|\mathbf{v}\|_{0,\Omega}^2+\|\text{curl}\mathbf{v}\|_{0,\Omega}^2\}^{1/2}.$$

The variational formulation based on (1)-(4) reads as: find $(\mathbf{E},H)\in\mathbf{H}_0(\text{curl};\Omega)\times L^2(\Omega)$ such that

$$(\mathbf{E}_t,\mathbf{\Phi})-(H,\text{curl}\mathbf{\Phi})=-(\mathbf{J},\mathbf{\Phi})\quad\forall\mathbf{\Phi}\in\mathbf{H}_0(\text{curl};\Omega),\tag{5}$$

$$(H_t,\psi)+(\text{curl}\mathbf{E},\psi)=0\qquad\forall\psi\in L^2(\Omega),\tag{6}$$

$$\mathbf{E}(0)=\mathbf{E}_0,\ \ H(0)=H_0.\tag{7}$$

Let T_h be a regular partition of Ω and $\mathbf{V}_h\times W_h\subset\mathbf{H}_0(\text{curl};\Omega)\times L^2(\Omega)$ be the finite element space. Then the finite element approximation based on (5)-(7) reads as follows ([7]): find $(\mathbf{E}_h,H_h)\in\mathbf{V}_h\times W_h$ such that

$$((\mathbf{E}_h)_t,\mathbf{\Phi})-(H_h,\text{curl}\mathbf{\Phi})=-(\mathbf{J},\mathbf{\Phi})\quad\forall\mathbf{\Phi}\in\mathbf{V}_h,\tag{8}$$

$$((H_h)_t,\psi)+(\text{curl}\mathbf{E}_h,\psi)=0\qquad\forall\psi\in W_h,\tag{9}$$

$$\mathbf{E_h}(0)=R_h\mathbf{E}_0,\ \ H(0)=R_hH_0,\tag{10}$$

where R_h is the mixed elliptic projection which is given by (22)-(25). Since (8)-(10) is an ordinary differential equations with respect to time t, there exists a unique solution. In this paper, we will consider the k-th ($k\geq 1$) Nedelec finite element spaces [9].

3. Nedelec Finite Element Spaces

Let Ω be a polygon with boundaries parallel to the axes. $T_h=\{e\}$ is a rectangulation of Ω, where

$$e=[x_e-h_e,x_e+h_e]\times[y_e-k_e,y_e+k_e]$$

and $h=\max_e\{h_e,k_e\}$. T_h is called regular if

$$C_0h^2\leq\text{meas}(e)\leq C_1h^2\quad\forall e\in T_h.$$

The Nedelec finite element spaces come from Raviart-Thomas finite element spaces [10]. We first list some properties on these two finite element spaces.

Raviart-Thomas finite element spaces $(\mathbf{V}_1)_h\times W_h$ is defined by

$$(\mathbf{V}_1)_h=\{\mathbf{v}=(v_1,v_2)\in\mathbf{H}_0(\text{div};\Omega);\mathbf{v}\mid_e\in Q_{k+1,k}(e)\times Q_{k,k+1}(e),e\in T_h\},\tag{11}$$

$$W_h=\{w\in L^2(\Omega);w\mid_e\in Q_k(e),e\in T_h\},\tag{12}$$

where

$$\mathbf{H}_0(\text{div};\Omega)=\left\{\mathbf{v}\in L^2(\Omega)^2;\text{div}\mathbf{v}=\frac{\partial v_1}{\partial x}+\frac{\partial v_2}{\partial y}\in L^2(\Omega),\mathbf{v}\cdot\mathbf{n}\mid_{\partial\Omega}=0\right\}$$

with norm

$$\|\mathbf{v}\|_{\mathbf{H}(\text{div};\Omega)}=\{\|\mathbf{v}\|_{0,\Omega}+\|\text{div}\|_{0,\Omega}^2\}^{1/2},$$