Journal of Computational Mathematics, Vol.22, No.1, 2004, 137–144.

A KIND OF MULTIVARIATE NURBS SURFACES *1)

Ren-Hong Wang   Chong-Jun Li
(*Institute of Mathematical Sciences, Dalian University of Technology, Dalian 116024, China*)

Abstract

The purpose of this paper is to construct a kind of multivariate NURBS surfaces by using the bivariate B-splines in the space $S_2^1(\triangle_{mn}^{(2)})$ and discuss some properties of this kind of NURBS surfaces with multiple knots on the type-2 triangulation.

Key words: Bivariate B-spline, Type-2 triangulation ($\triangle_{mn}^{(2)}$), NURBS surfaces.

1. Introduction

As we know that the NURBS surface was usually obtained by using the tensor product B-splines. For example, the bi-quadratic B-spline surface

$$\mathbf{S}(u,v)=\sum_{i=0}^{m}\sum_{j=0}^{n}N_{i,2}(u)N_{j,2}(v)\mathbf{P}_{i,j}, \tag{1}$$

in general, is a surface of degree 2 in the u or v direction. However, it should be a surface of degree 4 in other ways. As a result, there may be some inflection points on the surface. In some cases, we would prefer quadratic surfaces (or piecewise quadratic surfaces) to the tensor product surfaces. In order to resolve the problem, we can construct another kind of NURBS surfaces by using the local supported bivariate B-splines in the space $S_2^1(\triangle_{mn}^{(2)})$ because they span the whole space as proved in (Ren-Hong Wang, 1985; Ren-Hong Wang, 1994). Moreover, since the basis of B-splines satisfies the partition of unity, these surfaces have stronger convex hull property and transformation invariance. After presenting the definition of the bivariate NURBS surface, section 2 shows some properties of this kind of NURBS surfaces with multiple knots on the type-2 triangulation, and presents several examples to demonstrate these properties.

2. Construction of Surface

2.1. Parametric Representation

* Received December 17, 2001.
1) Project supported by the National Natural Science Foundation of China (No.69973010, No.10271022, No.60373093), and the Guangdong Natural Science Foundation, China (No.021755).

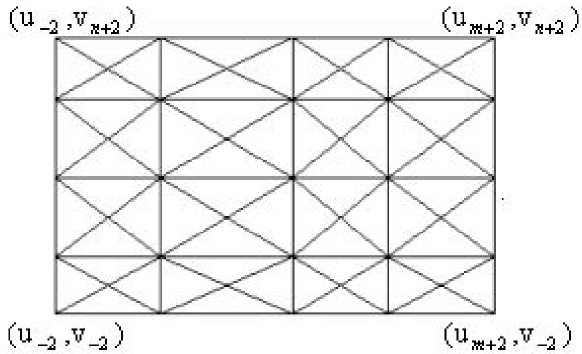

Figure 1. A type-2 triangulation of parametric domain.

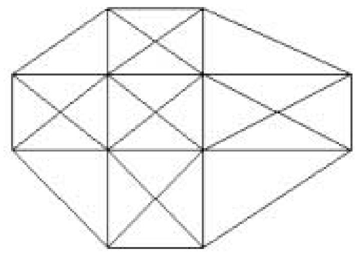

Figure 2. The support of B-spline.

Let $\mathbf{c}_{ij} \in R^3$ be control points, and $w_{ij} \in R$ be weights $(i = -1, 0, \cdots, m; j = -1, 0, \cdots, n)$. By using appropriate parameters u and v, we obtain a type-2 triangulation shown in Figure 1. In this paper we consider only control points corresponding to those intersection points of two diagonals in each subrectangle. The support of B-spline is shown in Figure 2, so the parametric knots should be:

$$\begin{cases} u_{-2} \le u_{-1} \le u_0 \le \cdots \le u_m \le u_{m+1} \le u_{m+2} \\ v_{-2} \le v_{-1} \le v_0 \le \cdots \le v_n \le v_{n+1} \le v_{n+2} \end{cases}.$$

Define non-uniform B-splines $B_{ij}(u,v) \in S_2^1(\triangle_{mn}^{(2)})$ which are shown in appendix, $(u,v) \in [u_{i-1}, u_{i+2}] \times [v_{j-1}, v_{j+2}], i = -1, 0, \cdots, m; j = -1, 0, \cdots, n$(Ren-Hong Wang, 1994). Then the bivariate NURBS surface is defined by:

$$\mathbf{S}(u,v) = \frac{\sum_{i=-1}^{m} \sum_{j=-1}^{n} \mathbf{c}_{ij} w_{ij} B_{ij}(u,v)}{\sum_{i=-1}^{m} \sum_{j=-1}^{n} w_{ij} B_{ij}(u,v)}, \quad (u,v) \in [u_0, u_m] \times [v_0, v_n]. \tag{2}$$

2.2. Uniform B-spline Surface

Let $\Delta u_i = u_i - u_{i-1} = a$, $\Delta v_j = v_j - v_{j-1} = b$, $i = -1, 0, \cdots, m+2$; $j = -1, 0, \cdots, n+2$, $a, b \in R$. Then $B_{ij}(u,v) = B_{00}(u - ia, v - jb)$, $i = -1, 0, \cdots, m$; $j = -1, 0, \cdots, n$, and a uniform B-spline surface is defined by:

$$\mathbf{S}(u,v) = \sum_{i=-1}^{m} \sum_{j=-1}^{n} \mathbf{c}_{ij} B_{ij}(u,v), \quad (u,v) \in [u_0, u_m] \times [v_0, v_n]. \tag{3}$$