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APPLICATION OF HOMOTOPY METHODS TO POWER SYSTEMS*[1)]

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Abstract

In this paper, the application of homotopy methods to the load flow multi-solution problems of power systems is introduced. By the generalized Bernshtein theorem, the combinatorial number C_{2n}^{n} is shown to be the BKK bound of the number of isolated solutions of the polynomial system transformed from load flow equations with generically chosen coefficients. As a result of the general Bezout number, the number of paths being followed is reduced significantly in the practical load flow computation. Finally, the complete P-V cures are obtained by tracking the load flow with homotopy methods.

Key words: Homotopy methods, Bezout number, Bernshtein-Khoranski-Kushnirenko (BKK) bound, Load flow computations.

1. Introduction

Load flow computations play an important role in the power system analysis([1], [2], [3]). In many cases, it comes down to solve a system of nonlinear algebraic equations with some constraint conditions. The algorithms available at present for this problem, such as Newton's methods and its variations , can be used to obtain individual solution only if the initial guess is in the near neighborhood of a solution. However, all solutions must be computed to explore the mechanism of voltage instability and collapse. Many power scientists and mathematicians have been very interested in this problem. It's well-known that there are few other general methods for determining all the solutions of nonlinear algebraic equations except homotopy methods ([4], [5], [6], [7]), which requires large amounts of computer time because of that many redundant paths must be followed during the computation.

This paper investigates homotopy methods and their numerical implementation for load flow multi-solution problems of power systems. The results significantly reduce the number of paths being followed. Since homotopy methods can calculate all isolated solutions of nonlinear algebraic equations in theory, they also are a benchmark for algorithmic development in this field.

2. Mathematical Model of Load Flow

Consider an $n+1$ bus (not including the earth bus) power system with r PQ buses, $n-r$ PV buses, and a slack bus. Since the computed results are independent of the ordering of the buses, without loss of generality, we assume that the previous r buses are PQ ones followed by

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PV buses and the last one is slack bus. From Kirchhoff law, we have a system of nonlinear complex-valued algebraic equations for the bus voltages $\dot{U}_1, \ldots, \dot{U}_n$,

$$P_i - \mathbf{j}\, Q_i = \overset{*}{\dot{U}_i} \sum_{j=1}^{n+1} Y_{ij} \dot{U}_j,\ i = 1, \ldots, n, \tag{1}$$

where both the slack bus voltage $\dot{U}_{n+1}$ and the admittance matrix $Y = (Y_{ij})_{(n+1)\times(n+1)}$ are known. The real and reactive bus powers $P_i, Q_i (i = 1, \ldots, r)$ on the PQ buses are also given, so that only the r bus voltages $\dot{U}_i (i = 1, \ldots, r)$ are unknown. On the other hand, the real bus powers and voltage peaks $P_i, V_i (i = r + 1, \ldots, n)$ on the PV buses are given as well, only the bus reactive powers and bus angles $Q_i, \delta_i (i = r + 1, \ldots, n)$ are expected. From the bus voltage peaks on the PV buses, we can also have $n - r$ additional equations

$$\dot{U}_i \overset{*}{\dot{U}_i} = V_i,\ i = r + 1, \ldots, n. \tag{2}$$

Equations (1) and (2) determine all the $2n - r$ unknowns.

For the sake of symbolic simplicity, Equations (1) and (2) can be rewritten in the following form

$$\begin{cases} \bar{x}_i (b_{i1} x_1 + \ldots + b_{in} x_n + d_i) - w_i = 0, & i = 1, \ldots, n, \\ \bar{x}_i x_i = V_i^2, & i = r + 1, \ldots, n. \end{cases} \tag{3}$$

where $x_i = \dot{U}_i \in \mathbf{C}$, "-" denotes the conjugate operation, $B = (b_{ij})_{n\times n} = (Y_{ij})_{n\times n}$, $d_i = Y_{i,n+1} \dot{U}_{n+1}$, $w_i = P_i - \mathbf{j}\, Q_i$ and B is a complex symmetrical matrix which is a highly sparse one when n is very large. The solutions of practical interests always have constraint conditions on the bus powers and voltages. However, the first phase of the algorithm dose not consider the constraint conditions.

3. Homotopy Methods for Load Flow Problems

Equation (3) can not be directly solved by homotopy methods because it is not a traditional polynomial equation system in complex variables. Substituting $z_i = x_i, z_{n+i} = \bar{x}_i (i = 1, \ldots, n)$ into Eq. (3) and conjugating every equation and eliminating the unknown reactive powers at the PV buses give the following equation system

$$\begin{cases} z_{n+i}(b_{i1} z_1 + \ldots + b_{in} z_n + d_i) - w_i = 0, & i = 1, \ldots, r, \\ z_i (b_{i1} z_{n+1} + \ldots + b_{in} z_{2n} + d_i) - \bar{w}_i = 0, & i = 1, \ldots, r, \\ z_{n+r+i}(\sum_{j=1}^{n} b_{r+i,j} z_j + d_{r+i}) + z_{r+i}(\sum_{j=1}^{n} \bar{b}_{r+i,j} z_{n+j} + \bar{d}_{r+i}) & \\ -2P_{r+i} = 0, & i = 1, \ldots, n - r, \\ z_{r+i} z_{n+r+i} = V_{r+i}^2, & i = 1, \ldots, n - r. \end{cases} \tag{4}$$

This is a system of $2n$ quadratic polynomial equations for the complex variables $\{z_1, \ldots, z_{2n}\}$ with complex coefficients, which can be as denoted $P(z) = 0$ where $z \in \mathbf{C}^{2n}$. Equation (3) can be easily proved to be equivalent to

$$\begin{cases} P(z) = 0; \\ z_i = \bar{z}_{n+i}, \quad i = 1, \ldots, n. \end{cases} \tag{5}$$

for determining the bus voltages. Thus, $P(z) = 0$ can be firstly solved, then the result is checked to see if $z_i = \bar{z}_{n+i} (i = 1, \ldots, n)$, and finally all solutions of the bus voltages are obtained.