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NUMERICAL METHODS FOR MAXWELL'S EQUATIONS IN INHOMOGENEOUS MEDIA WITH MATERIAL INTERFACES *1)

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Dedicated to Professor Zhong-ci Shi on the occasion of his 70th birthday

Abstract

In this paper, we will present some recent results on developing numerical methods for solving Maxwell's equations in inhomogeneous media with material interfaces. First, we will present a second order upwinding embedded boundary method - a Cartesian grid based finite difference method with special upwinding treatment near the material interfaces. Second, we will present a high order discontinuous spectral element with Dubinar orthogonal polynomials on triangles. Numerical results on electromagnetic scattering and photonic waveguide will be included.

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1. Introduction

Time domain solutions of Maxwell's equations have found applications in engineering problems such as designs of VLSI chips and photonic devices [1]. In contrast to frequency domain approaches where time harmonic Maxwell's equations are solved for given frequencies [2], the solutions from time domain simulation can produce a wide range of frequency information as well as transient phenomena required in many applications.

The most used time domain algorithm for Maxwell's equations is the simple Yee's finite difference scheme [3], which yields a second order approximation to the fields provided the underlying grids are rectangles and the conductor or dielectric boundaries are aligning with the mesh coordinates. Thus, the major disadvantage of the Yee's scheme is the limitation of the boundary or material interface geometry. To have second order accuracy, the scheme demands a locally conforming mesh to the boundary, as a result, tiny finite difference cells may limit the time step of the overall scheme.

Meanwhile, discontinuous Galerkin methods have attracted much research to handle the material interfaces in the media. Being higher order versions of traditional finite volume method [4], discontinuous Galerkin methods have been developed initially in 1970's for the study of neutron transport equations [5], and have now been applied to the area of computational fluid dynamics and the solution of Maxwell's equations [6] [7]. Discontinuous Galerkin methods inherit the flexibility of the finite element method in allowing unstructured meshes, and at the same time, employ high order polynomials for better accuracy and phase error in modelling wave propagations.

In this paper, we will first present a new upwinding embedded boundary method which employs a simple Cartesian grid to solve time dependent Maxwell's equations. The proposed

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embedded boundary method, like the immersed interface method (IIM) proposed to solve elliptic PDEs with discontinuous coefficients [8], uses a central difference scheme for mesh points away from the interfaces while modifications are made for grid points near the interfaces. Second, we will study a high order discontinuous spectral element with Dubinar orthogonal polynomials on triangles and Legendre orthogonal polynomials on quadrilaterals.

Numerical Results on electromagnetic scattering will be given for the upwinding embedded boundary methods while photonic waveguide with whispering gallery modes in microcylinders will be simulated with the discontinuous spectral element methods.

2. Upwinding Embedded Boundary Method

2.1. One Dimensional Scalar Model Equation

We will consider the following simple linear wave equation to demonstrate the basic idea of the upwinding embedded boundary method,

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0, \qquad 0 \le x \le 1, \tag{2.1}$$

where the wave speed a is assumed to be positive and discontinuous at $x_d \in (0,1)$, and the solution $u(x,t)$ satisfies a jump condition at x_d,

$$r^+ u(x_d^+,.) - r^- u(x_d^-,.) = g. \tag{2.2}$$

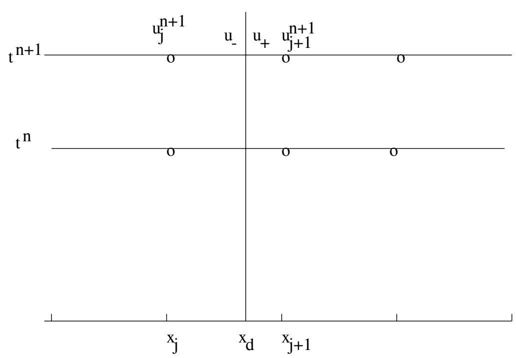

Figure 1: 1-D mesh with discontinuity at x_d

For a uniform grid $\{x_i = i\Delta x,\ 0 \le i \le N,\ \Delta x = \frac{1}{N}\}$, we have the numerical solutions u_i^n at grid points $(x_i,\ t^n), i = 0, 1, \cdots, N$, and also the solutions at both sides of the jump location x_d denoted as u_-^n, u_+^n (see Figure 1). Let us assume that $x_d \in [x_j, x_{j+1}]$,and $x_d = x_j + \alpha\Delta x$, $x_{j+1} - x_d = \beta\Delta x$, where $\alpha + \beta = 1$.

We will construct a uniformly second order finite difference method to solve (2.1) based on the Lax-Wendroff approach

$$u^{n+1} \doteq u^n + \Delta t u_t^n + \frac{(\Delta t)^2}{2} u_{tt}^n = u^n - a\Delta t u_x^n + \frac{(a\Delta t)^2}{2} u_{xx}^n, \tag{2.3}$$

where $\Delta t = CFL\frac{\Delta x}{|a|}$, and the spatial derivatives can be approximated by appropriate finite differences Let us assume that the solutions $u_i^n, 0 \le i \le N$, u_-^n and u_+^n have been obtained for the time step $t = t^n$. We will show how to obtain the solutions at the time step $t = t^{n+1}$.

- Solutions at the jump x_d