Journal of Computational Mathematics, Vol.22, No.3, 2004, 475–488.

MORTAR FINITE VOLUME METHOD WITH ADINI ELEMENT FOR BIHARMONIC PROBLEM [*1)]

Chun-jia Bi
(*Department of Mathematics, Yantai University, Yantai 264005, China*)

Li-kang Li
(*Department of Mathematics, Fudan University, Shanghai 200433, China*)

Abstract

In this paper, we construct and analyse a mortar finite volume method for the discretization for the biharmonic problem in R^2. This method is based on the mortar-type Adini nonconforming finite element spaces. The optimal order H^2-seminorm error estimate between the exact solution and the mortar Adini finite volume solution of the biharmonic equation is established.

Mathematics subject classification: 65N30, 65N15.
Key words: Mortar finite volume method, Adini element, Biharmonic problem.

1. Introduction

In recent years, the mortar finite element method as a special nonconforming domain decomposition technique has attracted many researchers' attention. More and more papers on this method have appeared. We refer to [3] and [19] for the general presentation of the mortar element method and [2], [7],[12], [15], and [20] for details.

In the mortar finite element method, the computational domain is first decomposed into a coarse sub-domain partition. The triangulations on different sub-domains need not match across sub-domain interfaces. The basic idea of this method is to replace the strong continuity condition by a weaker suitable constraint on the interfaces between different sub-domains. Suitable constraint, i.e., the mortar condition, guarantees the optimal discretization schemes.

On the other hand, the finite volume method (also called the box method, generalized difference method) is popular in computational fluid mechanics due to their conservation properties of the original problems. In the past several decades, many researchers have analysed the finite volume method for the selfadjoint (or non-selfadjoint and nondefinite) elliptic partial differential equations using the finite element spaces. Professors Ronghua Li *et al* have systematically studied the finite volume method and obtained many significant results, we refer to the monograph [18] for the general presentation of the finite volume method and [1], [5] [6], [8], [9] [13], [16], [17], [21], and [22] for details.

Recently, Ewing, Lazarov and Lin [11] consider the mortar finite volume element approximations of second order elliptic equations on non-matching grids. The discretization is based on the Petrov-Galerkin method with a solution space of continuous piecewise linear functions over each sub-domain and a test space of piecewise constant functions. They use finite volume element approximations on the sub-domains and finite element on the interfaces for Lagrange multipliers and get an optimal order convergence in energy norm.

In the paper [14], we extend the mortar finite element method to the mortar finite volume method, construct and study a mortar finite volume method which is based on the mortar

[*] Received March 7, 2002; final revised September 9, 2003.
[1)] This research is supported by the National Natural Science Foundation of China under grant 10071015.

Crouzeix-Raviart finite element space. The optimal order error estimates in broken H^1-norm and in L^2-norm have been developed.

In this paper, we construct and analyse the mortar finite volume method with Adini nonconforming element which is used to solve the biharmonic problem. The restriction of the mortar finite element space to any sub-domain is the Adini nonconforming finite element space. In this paper, we will prove the optimal order error estimate in broken H^2-seminorm.

The remainder of this paper is organized as follows. In Section 2 we introduce notation, construct a triangulation $\mathcal{T}_h$ of Ω and give the corresponding dual partition. In Section 3, we consider the mortar finite volume method, and get some lemmas which will be used in later convergence proof. In Section 4, we estimate the difference between the exact solution and the mortar finite volume approximation in H^2 broken seminorm.

2. Notation and Preliminaries

In this section, we provide some preliminaries and notation. In this paper, we suppose the boundary of the multi-rectangular domain Ω parallel to the OX_1 and OX_2 axises. Consider a geometrically conforming version of the mortar finite volume method, i.e., Ω is divided into non-overlapping rectangular sub-domains Ω_i

$$\overline{\Omega} = \cup_{i=1}^{N} \overline{\Omega}_i,$$

where $\overline{\Omega}_i \cap \overline{\Omega}_j$ is an empty set or a vertex or an edge for $i \neq j$.

Each sub-domain Ω_i is triangulated to produce an rectangular quasi-uniform mesh $\mathcal{T}_{h_i} = \{K\}$ with mesh parameter h_i, where h_i is the largest diameter of the elements in $\mathcal{T}_{h_i}$. The triangulations of sub-domains generally do not align at the sub-domain interfaces. Let Γ_{ij} denote the open straight line segment which is common to Ω_i and Ω_j and Γ denote the union of all interfaces between the sub-domains, i.e., $\Gamma = \cup \partial\Omega_i \backslash \partial\Omega$. We assume that the endpoints of each interface segment in Γ are vertices of $\mathcal{T}_{h_i}$ and $\mathcal{T}_{h_j}$. Let $\mathcal{T}_h$ denote the global mesh $\cup_i \mathcal{T}_{h_i}$ which is assumed quasi-uniform in this paper and $h = \max_{1 \le i \le N} h_i$.

Since the triangulation $\mathcal{T}_h$ is independent over the sub-domains, each side $\Gamma_{ij} = \overline{\Omega}_i \cap \overline{\Omega}_j$ is provided with two different and independent 1-D meshes, denoted by $\mathcal{T}_{h_i}(\Gamma_{ij})$ and $\mathcal{T}_{h_j}(\Gamma_{ij})$, respectively. We define one of the sides of Γ_{ij} as a mortar one, the other as a non-mortar one, which are denoted by γ_i and δ_j, respectively. The sets of vertices belonging to $\overline{\Omega}$, $\overline{\Omega}_i$, $\partial\Omega_i$, $\partial\Omega$, γ_i, δ_j and K are denoted by Ω_h, $\Omega_{i,h}$, $\partial\Omega_{i,h}$, $\partial\Omega_h$, $\gamma_{i,h}$, $\delta_{j,h}$ and K_h, respectively.

Define the Adini nonconforming finite element space on sub-domain Ω_i:

$$\begin{aligned} \widetilde{V}_{h,i} = \widetilde{V}_{h,i}(\Omega_i) = \{ & v \in L^2(\Omega_i) : v|_K \in P_3(K) \oplus \ \operatorname{span}\{x_1^3 x_2, x_1 x_2^3\} \ \text{ for } \ K \in \mathcal{T}_{h_i}, \\ & v, v_{x_1}, v_{x_2} \ \text{ are continuous at the vertices and} \\ & v(a) = v_{x_1}(a) = v_{x_2}(a) = 0, \ \ \forall a \in \partial\Omega_{i,h} \cap \partial\Omega_h \}. \end{aligned}$$

We can now introduce the global space $\widetilde{V}_h$:

$$\widetilde{V}_h = \prod_{i=1}^{N} \widetilde{V}_{h,i}(\Omega_i)$$

with the so called broken H^2-seminorm:

$$|v|_{2,h} = |v|_{2,h,\Omega} = \left(\sum_{i=1}^{N} |v|_{2,h,\Omega_i}^2 \right)^{\frac{1}{2}}, \quad |v|_{2,h_i,\Omega_i} = \left(\sum_{K \in \mathcal{T}_{h_i}} |v|_{H^2(K)}^2 \right)^{\frac{1}{2}}.$$

Let $W(\delta_j)$ be the subspace of the space $L^2(\Gamma_{ij})$:

$$W(\delta_j) = \{ v \in C^0(\overline{\delta}_j), v|_{\overline{K} \cap \delta_j} \in P_1(\overline{K} \cap \delta_j), \quad \forall K \in \mathcal{T}_{h_j} \}.$$