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A LINE SEARCH AND TRUST REGION ALGORITHM WITH TRUST REGION RADIUS CONVERGING TO ZERO *1)

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Abstract

In this paper, we present a new line search and trust region algorithm for unconstrained optimization problem with the trust region radius converging to zero. The new trust region algorithm performs a backtracking line search from the failed point instead of resolving the subproblem when the trial step results in an increase in the objective function. We show that the algorithm preserves the convergence properties of the traditional trust region algorithms. Numerical results are also given.

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Key words: Unconstrained optimization, Trust region, Line search.

1. Introduction

In this paper, we consider the line search and trust region method for the unconstrained optimization problem

$$\min_{x\in R^n} \ f(x), \tag{1.1}$$

where $f : R^n \to R$ is continuously differentiable. In every iteration, a trial step is computed by solving the subproblem

$$\begin{array}{ll} \min\limits_{d\in R^n} & g_k^T d + \dfrac{1}{2} d^T B_k d := \phi_k(d) \\ s.\ t. & \|d\| \le \Delta_k, \end{array} \tag{1.2}$$

where $g_k = \nabla f(x_k), B_k$ is a $n \times n$ symmetric matrix which approximates the Hessian of f at x_k, $\Delta_k > 0$ is the current trust region radius, and $||\cdot||$ refers to the 2-norm.

Let d_k be the solution of (1.2). The predicted reduction is defined by the reduction of the approximate model, that is,

$$Pred_k = \phi_k(0) - \phi_k(d_k), \tag{1.3}$$

and the actual reduction is defined by

$$Ared_k = f(x_k) - f(x_k + d_k). \tag{1.4}$$

The ratio between these two reductions is defined by

$$r_k = \frac{Ared_k}{Pred_k}, \tag{1.5}$$

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and it plays a key role in the traditional trust region algorithm (TTR) to decide whether the trial step is acceptable and to adjust the new trust region radius. If the trial step is not successful, then we reject it, reduce the trust region radius, and resolves the subproblem (1.2), otherwise, we accept the trial step, and enlarge the trust region radius. That is, in TTR, we choose:

$$x_{k+1} = \begin{cases} x_k + d_k, & \text{if } r_k > c_0, \\ x_k, & \text{otherwise,} \end{cases} \tag{1.6}$$

where $c_0 \in [0, 1)$ is a small constant, and choose

$$\Delta_{k+1} \in \begin{cases} [c_3||d_k||,\ c_4\Delta_k], & \text{if } r_k < c_2, \\ [\Delta_k,\ c_1\Delta_k] & \text{otherwise,} \end{cases} \tag{1.7}$$

where $0 < c_3 < c_4 < 1 < c_1, 0 \leq c_0 \leq c_2 < 1$ are constants.

In TTR, when the sequence $\{x_k\}$ converges to the minimizer x^* of the objective function f, the ratio of the actual reduction and the predicted reduction r_k will converge to 1. For the details of trust region algorithms, please see [8, 9, 10]. It then follows from the updating rule of the trust region radius (1.7) that Δ_k will be larger than a positive constant for all sufficiently large k, hence, the trust region will not play the role at the end. In fact, it suffices for the convergence that Δ_k be larger than $O(||x_k - x^*||)$ at every iteration. To prevent the trial step from being too large near the minimizer, we present a trust region algorithm for (1.1) with the trust region radius converging to zero [1]. We choose

$$\Delta_k = \mu_k ||g_k||, \tag{1.8}$$

where μ_k is updated according to the ratio r_k.

As we know, to resolve the subproblem (1.2) can be costly, since this requires solving one or more linear systems as follows:

$$(B_k + \lambda I)d = -g_k, \tag{1.9}$$

while line search methods require little computation to decide a new point. Nocedal and Yuan creatively combine the trust region technique and line search technique in [5]. In this paper, we apply the line search technique to our trust region algorithm with the trust region converging to zero. The subproblem is solved by the algorithm given by Nocedal & Yuan in [5], hence the trial step d_k is always a direction of sufficient descent for the objective function. Thus we do not need to resolve the subproblem (1.2) when $f(x_k + d_k) > f(x_k)$, in stead we can carry out the backtracking line search along d_k until we obtain the new trial point at which the value of the objective function is less than $f(x_k)$.

In the next section, we present the new line search and trust region algorithm with the trust region converging to zero, and show that the new algorithm preserves the global convergence of the traditional trust region algorithm. In section 3, we discuss the superlinear convergence of the algorithm. Finally in section 4, some numerical results are given.

2. The Algorithm and Global Convergence

In this section, we first give some properties of the trust region subproblem (1.2), then present our new line search and trust region algorithm with the trust region converging to zero, finally we discuss the global convergence of the new algorithm.

The following results are well known(see Moré and Sorensen [4] and Gay [2]).

Lemma 2.1. *A vector $d^* \in R^n$ is a solution of the problem*

$$\begin{array}{ll} \min\limits_{d \in R^n} & g^T d + \frac{1}{2} d^T B d := \phi(d) \\ s.t. & ||d|| \leq \Delta, \end{array} \tag{2.1}$$